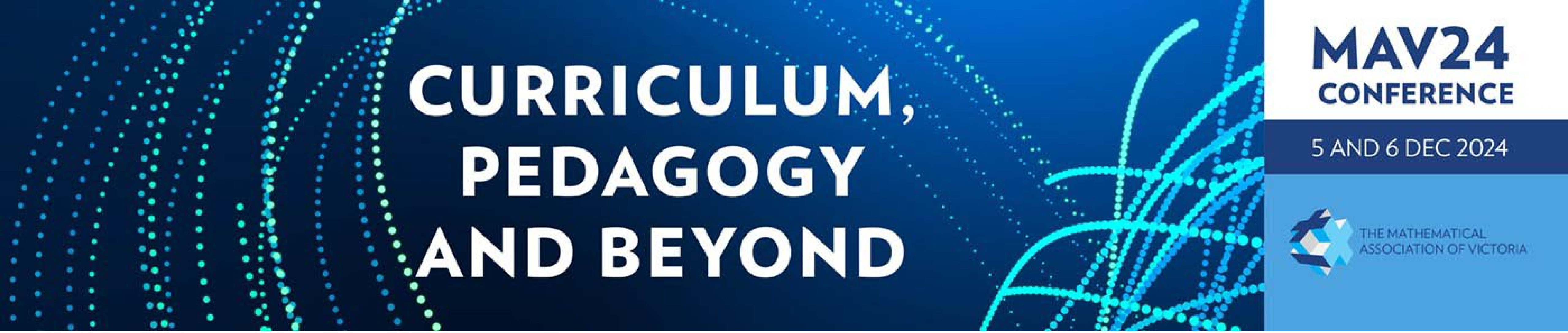


Real trigonometry using real-time, real-world data with the app *Flightradar24*

Presented by Enzo Vozzo, 5 & 6 Dec 2024

Mathematics Teacher at Mentone Grammar

H25 REAL TRIGONOMETRY USING REAL-TIME REAL-WORLD DATA

Subtheme: Technology

Enzo Vozzo, Mentone Grammar
(Year 9 to Year 12)

Flightradar24, a popular plane tracking app, gives users access to a flight's real time data such as speed, altitude, track, latitude and longitude. Using plane and spherical trigonometry, this real-time, real-world data can be used to calculate and confirm that the speed and track of a flight are correct using four different methods. Three methods involve plane trigonometry, and these will depend on particular aspects of a flight: Method 1 deals with flights that are travelling due north or south, Method 2 deals with flights that are travelling due east or west, Method 3 deals with flights near the equator travelling in any direction. Method 4 uses spherical trigonometry and is the method that is actually used by flights. The theory behind each method will be discussed along with worked examples. All these calculations can be done on a CAS calculator or on a spreadsheet.

Key takeaways:

1. A practical application of the use of trigonometry.

2. Great for students who want to be pilots.

3. The use of a spreadsheet and or CAS calculator to perform trigonometric navigation calculations.

Remember: Delegates should be familiar with the app Flightradar24 and have it installed on their mobile phone.

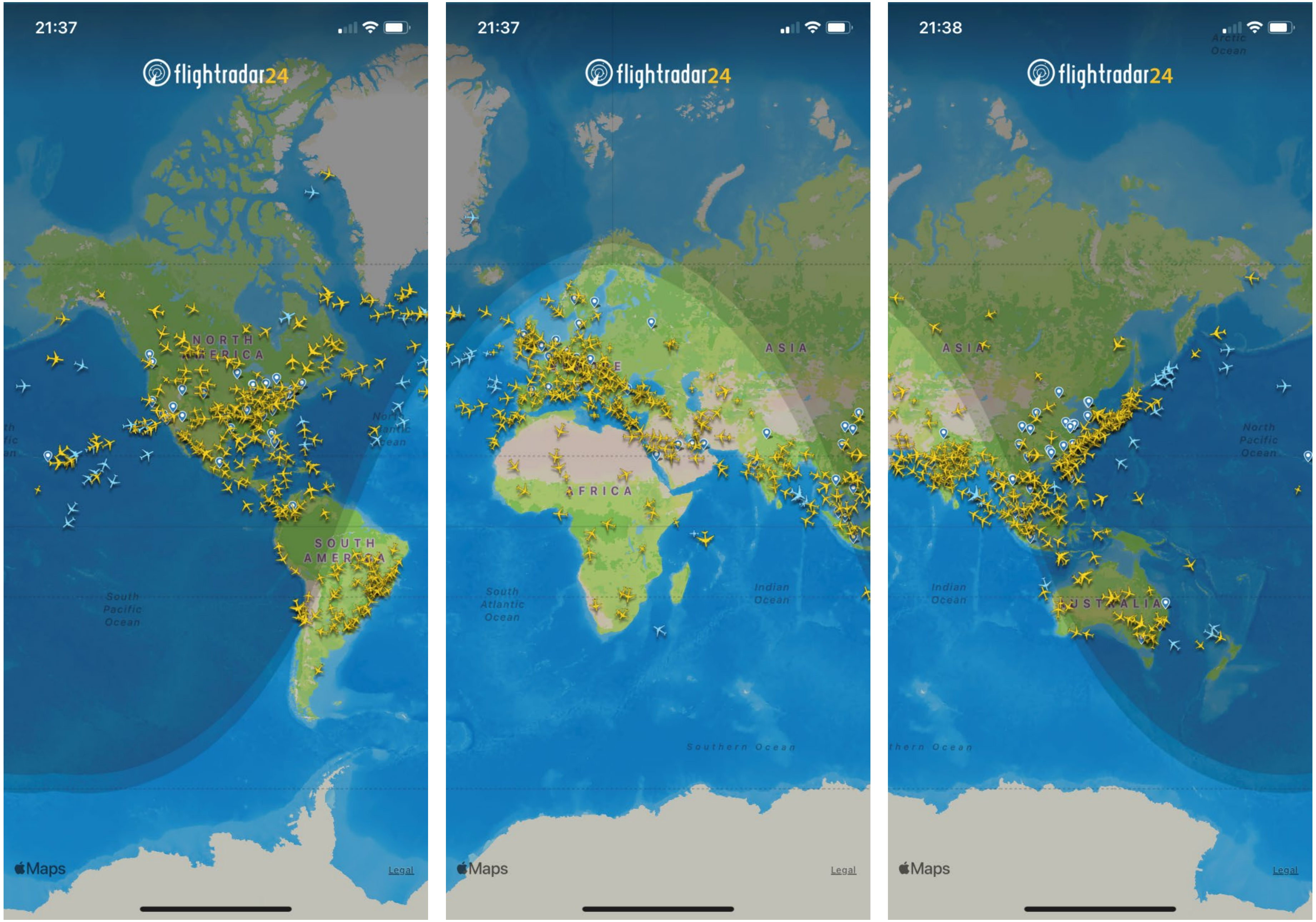
The app
Flightradar24
gives real time flight data on commercial flights around the world.

These data include:

- *Latitude in decimal degrees*
- *Longitude in decimal degrees*
- *Speed in knots (nm per hour)*
- $1\ knot = 1\ nautical\ mile/hour$
- $1\ nautical\ mile = 1.852\ km$
- *Altitude in feet*
- *Track (Bearing) in °True*

Flightradar24
App icon

Typical views

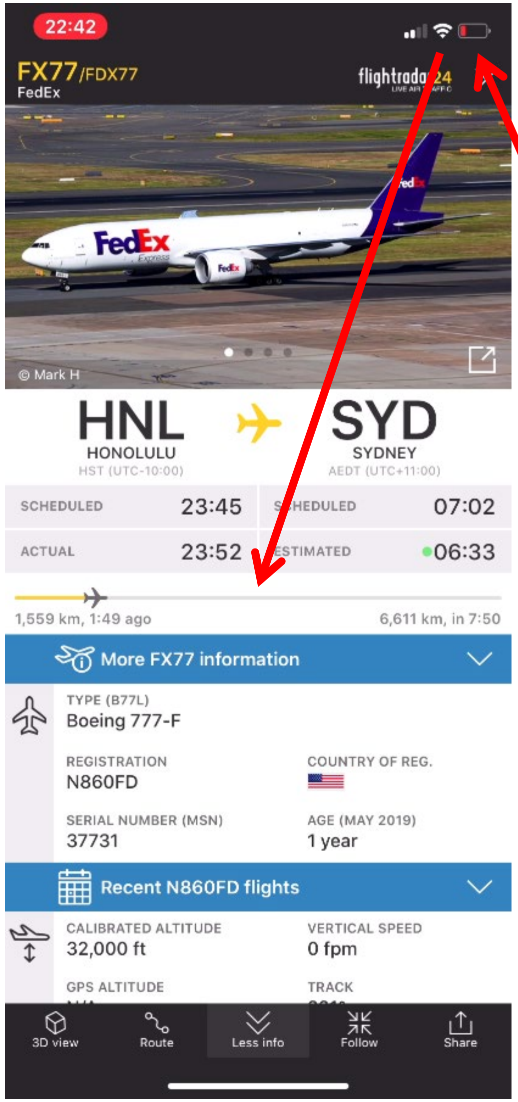


*The best way to capture these data is with a **screen recording** on your mobile phone.*

*During the screen recording, the speed and direction of travel **must** remain constant.*

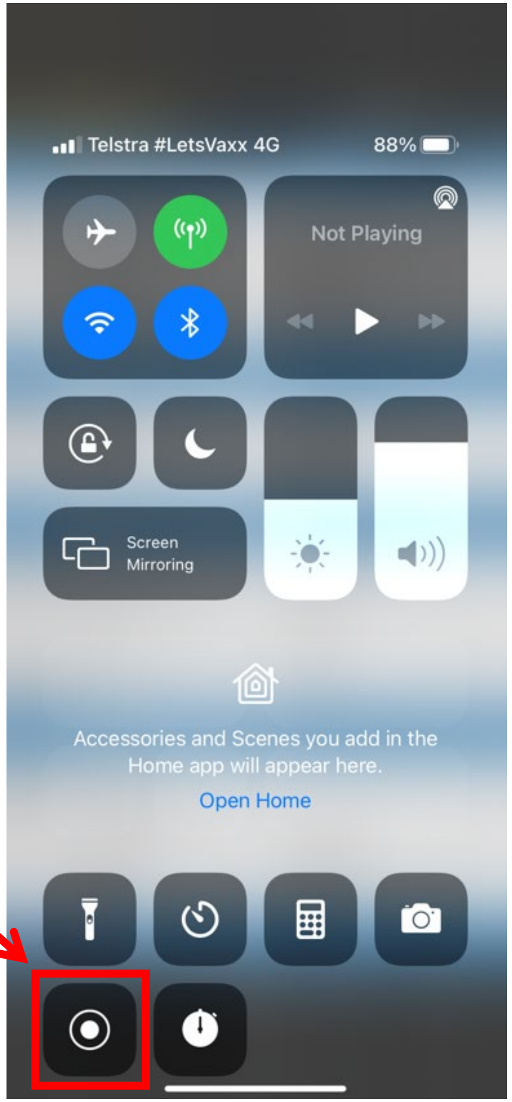
*Record **approximately** **100** **seconds or more** to minimise errors caused by the one second timing resolution and three decimal places of Latitude and Longitude.*

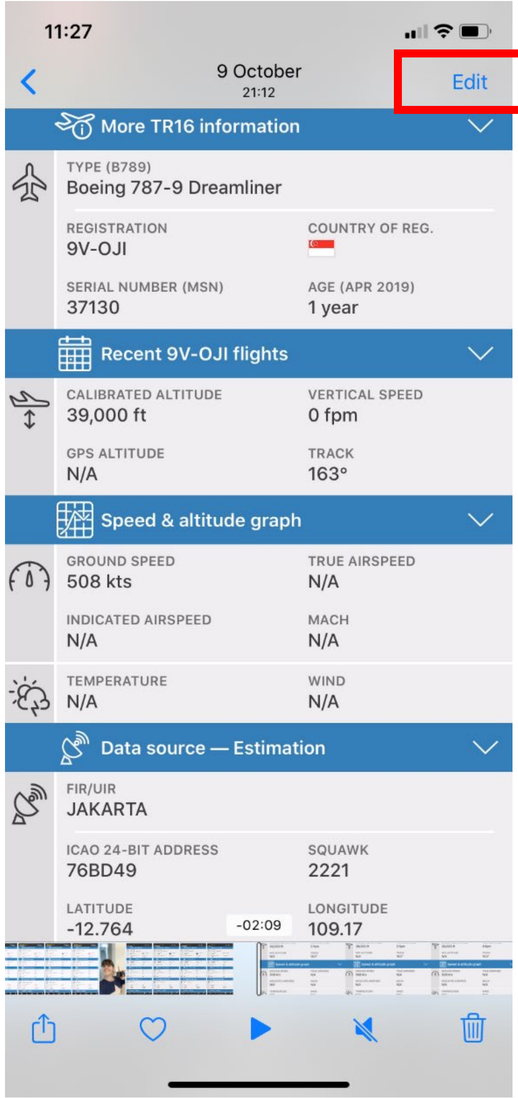
Recording a flight video using an iPhone 11 with iOS 16



Swipe down from here to start and end the video.

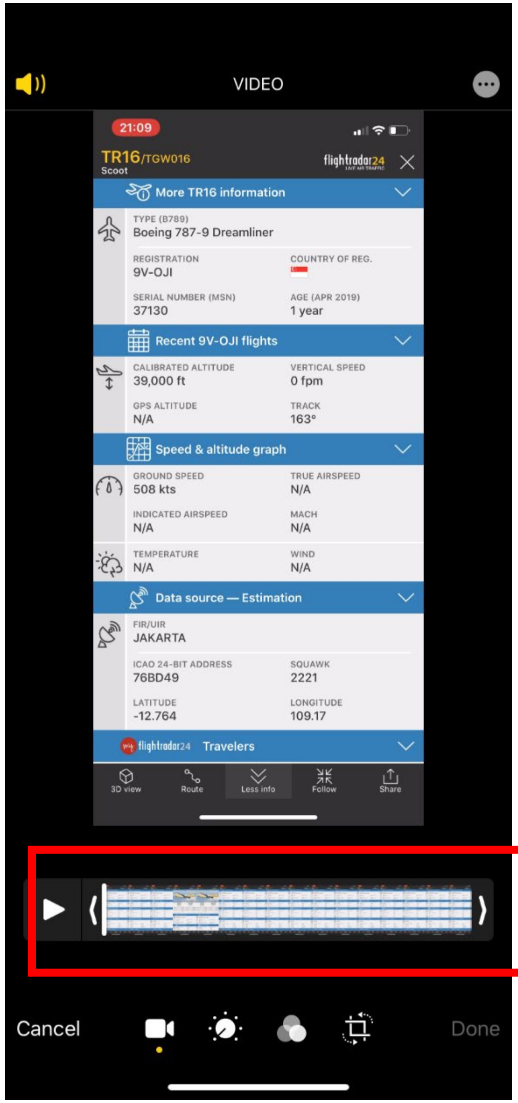
Screen record start and stop button





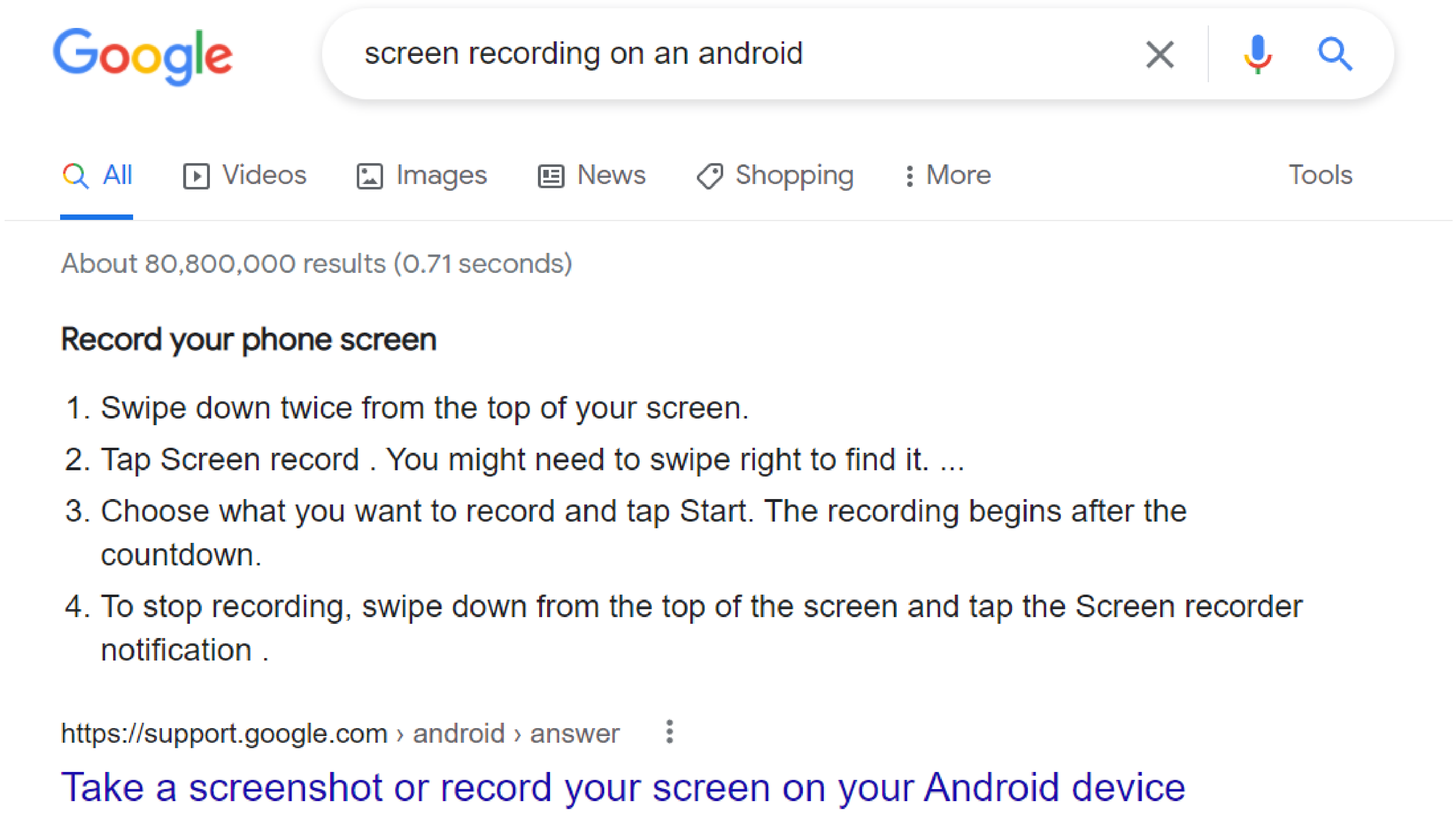
Edit video

To edit video, go to the iPhone's Photo Library. This is necessary because the screen recorder screen needs to be cut from the start and end of the video.

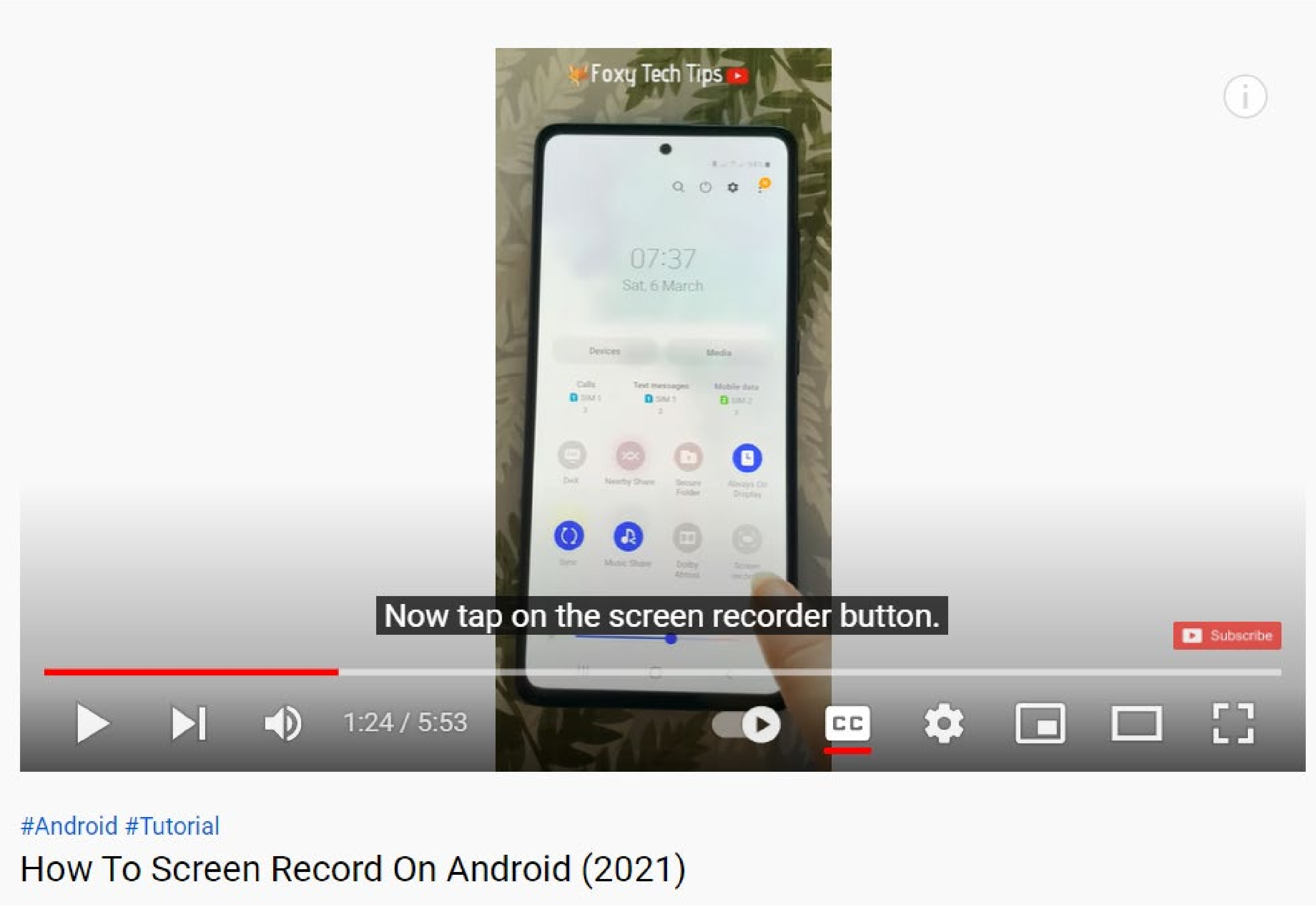


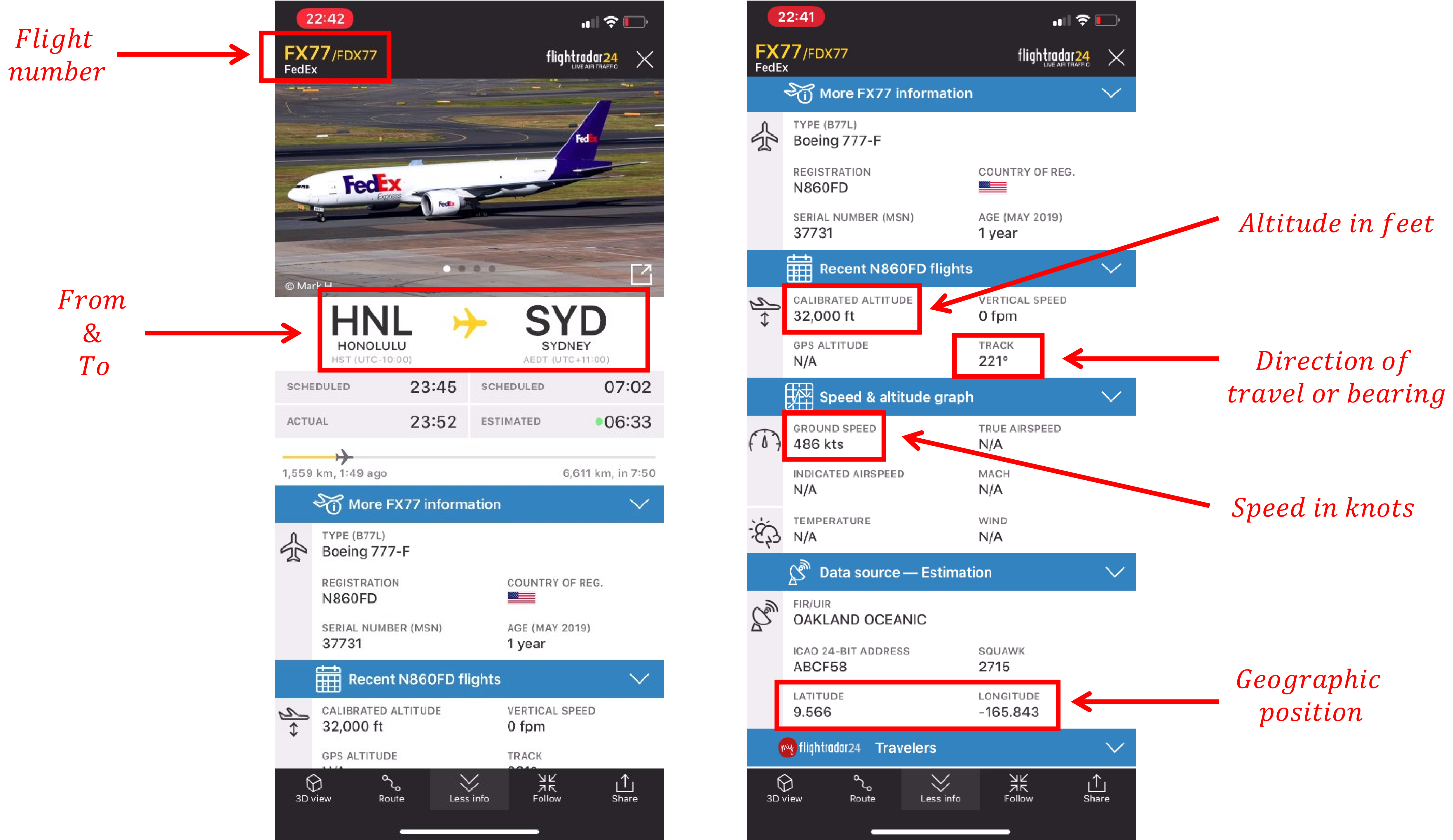
Edit video

Recording a flight video using an Android phone



https://www.youtube.com/watch?v=0vza_7zo_Mk





Flight number
From & To
Altitude in feet
Direction of travel or bearing
Speed in knots
Geographic position
22:42
FX77/FDX77
FedEx
flightradar24
LIVE AIR TRAFFIC
© Mark H
HNL
HONOLULU
HST (UTC-10:00)
SYD
SYDNEY
AEDT (UTC+11:00)
SCHEDULED 23:45
SCHEDULED 07:02
ACTUAL 23:52
ESTIMATED 06:33
1,559 km, 1:49 ago
6,611 km, in 7:50
More FX77 information
TYPE (B77L)
Boeing 777-F
REGISTRATION
N860FD
COUNTRY OF REG.
SERIAL NUMBER (MSN)
37731
AGE (MAY 2019)
1 year
Recent N860FD flights
CALIBRATED ALTITUDE
32,000 ft
VERTICAL SPEED
0 fpm
GPS ALTITUDE
TRACK
3D view
Route
Less info
Follow
Share
22:41
FX77/FDX77
FedEx
flightradar24
More FX77 information
TYPE (B77L)
Boeing 777-F
REGISTRATION
N860FD
COUNTRY OF REG.
SERIAL NUMBER (MSN)
37731
AGE (MAY 2019)
1 year
Recent N860FD flights
CALIBRATED ALTITUDE
32,000 ft
VERTICAL SPEED
0 fpm
GPS ALTITUDE
N/A
TRACK
221°
Speed & altitude graph
GROUND SPEED
486 kts
TRUE AIRSPEED
N/A
INDICATED AIRSPEED
N/A
MACH
N/A
TEMPERATURE
N/A
WIND
N/A
Data source — Estimation
FIR/UIR
OAKLAND OCEANIC
ICAO 24-BIT ADDRESS
ABCF58
SQUAWK
2715
LATITUDE
9.566
LONGITUDE
-165.843
flightradar24 Travelers
3D view
Route
Less info
Follow
Share

***This presentation is based on the article I published in the MAV journal* Vinculum, *Volume* 55, *Number* 4 *in* 2018.**

REAL TRIGONOMETRY WITH REAL TIME, REAL WORLD DATA

Today's technology allows public access to real time information at very low cost. One example of this comes from the aviation industry, which now allows public access to real time flight data from commercial airline flights. This position, speed, vertical speed, altitude and track (direction of travel or bearing) and can be accessed by websites and smartphone apps such as *Flightradar24*. This wealth of data is a goldmine for mathematics teachers and students who are studying trigonometry.

Mathematics is often seen as not relevant and so students do not engage with it. The data from *Flightradar24* can be used in classrooms to tackle the issue of relevancy and engage students. When students are first shown *Flightradar24* either through a website or smartphone app, many of them are amazed. What better 'hook' could be used to pique students' interest in a mathematics lesson? For those who dream of becoming a pilot, what a great way to bring the world of aviation closer to them.

When students become familiar with interpreting this data, they can be asked to find flights that are:

- fastest
- highest
- longest
- travelling due north, south, east or west
- crossing either the equator, the Greenwich meridian, the International Date Line (IDL) or a combination
- travelling closest to the North Pole.

This can be done as a game in class or set as a task over a one-week period. Students will probably observe certain flights occurring repeatedly. As evidence of their findings, students can be asked to take screenshots or screen recordings and to present these to the class.

This data can be used in a middle secondary school mathematics class in confirming that the speed and track (bearing) of a flight are correct based on a flight's geographical position (latitude and longitude) given the elapsed time between the two positions. *Flightradar24* gives these coordinates in decimal degrees to four decimal places from −180° to 180° for longitude and −90° to 90° for latitude, enabling distances to be calculated. Knowing elapsed times allows speeds to be calculated. Obtaining the requisite data is best done on a smartphone using a screen recording and playing the recording to note the positions and the elapsed time.

There are four methods in which the distance, speed and track of a flight during a time interval can be calculated. Three are relatively simple using only plane trigonometry. One is more challenging using spherical trigonometry. In order of difficulty:

1. the track (bearing) is due north or south (no trigonometry required)
2. the track is due east or west
3. any track within a few degrees north or south of the equator (approximating to plane geometry)
4. any other flight (spherical trigonometry required).

In all these methods, the most difficult calculation is the distance travelled during the time interval. Whichever method is used, it is vital that the flight does not alter its speed or track during the time interval, as the calculations assume a constant speed and track. A typical time interval would be in the order of at least a few minutes. This is because the time indicator on a screen recording video is in seconds and so having a time interval of at least 100 seconds would minimise any error caused by a one-second time interval uncertainty. The longer the time interval, the greater the accuracy of the speed and track. Each of these four methods requires a different approach to calculating the distance travelled during the recorded time interval. The first three methods are well within the understanding of middle secondary school students, while the fourth method will challenge students.

DUE NORTH OR SOUTH

In this case, the track follows a meridian of longitude and all meridian lines are great circles. This greatly simplifies the calculation of the distance travelled during the elapsed time interval. Distance, speed and track are easily calculated.

$$\text{Distance} = \frac{\text{difference in latitude}}{360^\circ} \times 40\,000 \text{ km}$$

$$\text{Speed} = \frac{\text{distance}}{\text{time}} \text{ (using appropriate units)}$$

Track = 000°T (north) or 180°T (south)

40 000 km is the circumference of the Earth. Alternately, the circumference can be calculated from the Earth's mean radius of 6371 km. Either figures will give very accurate results.

Since the equator is also a great circle, this calculation can also be used for flights along the equator.

DUE EAST OR WEST

In this case, the flight's track follows a parallel of latitude (parallel to the equator). The radius of a circle of latitude is less than the Earth's radius, and can be found by multiplying the Earth's radius by the cosine of the angle of latitude, usually denoted by the Greek letter ϕ.

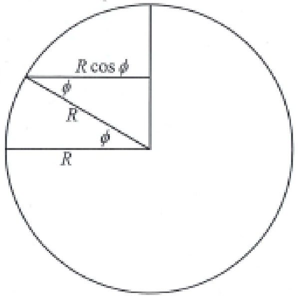
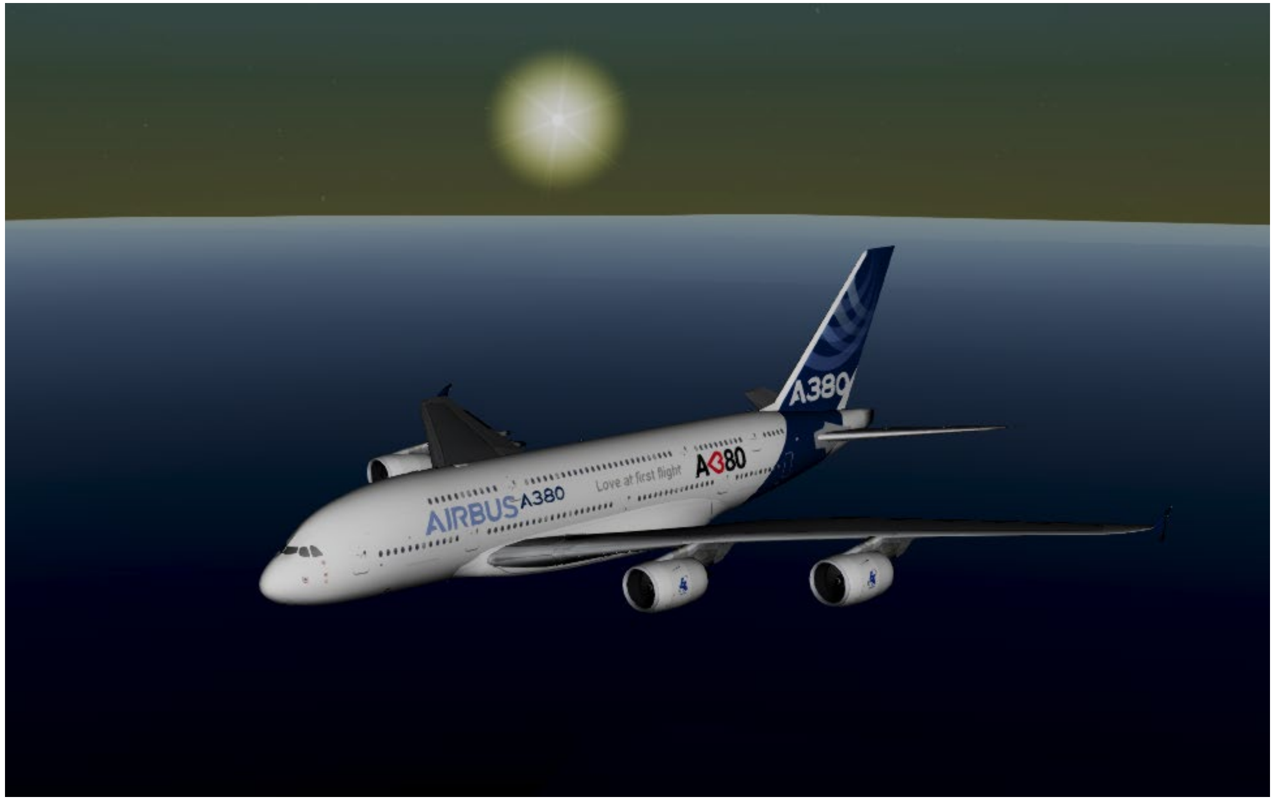


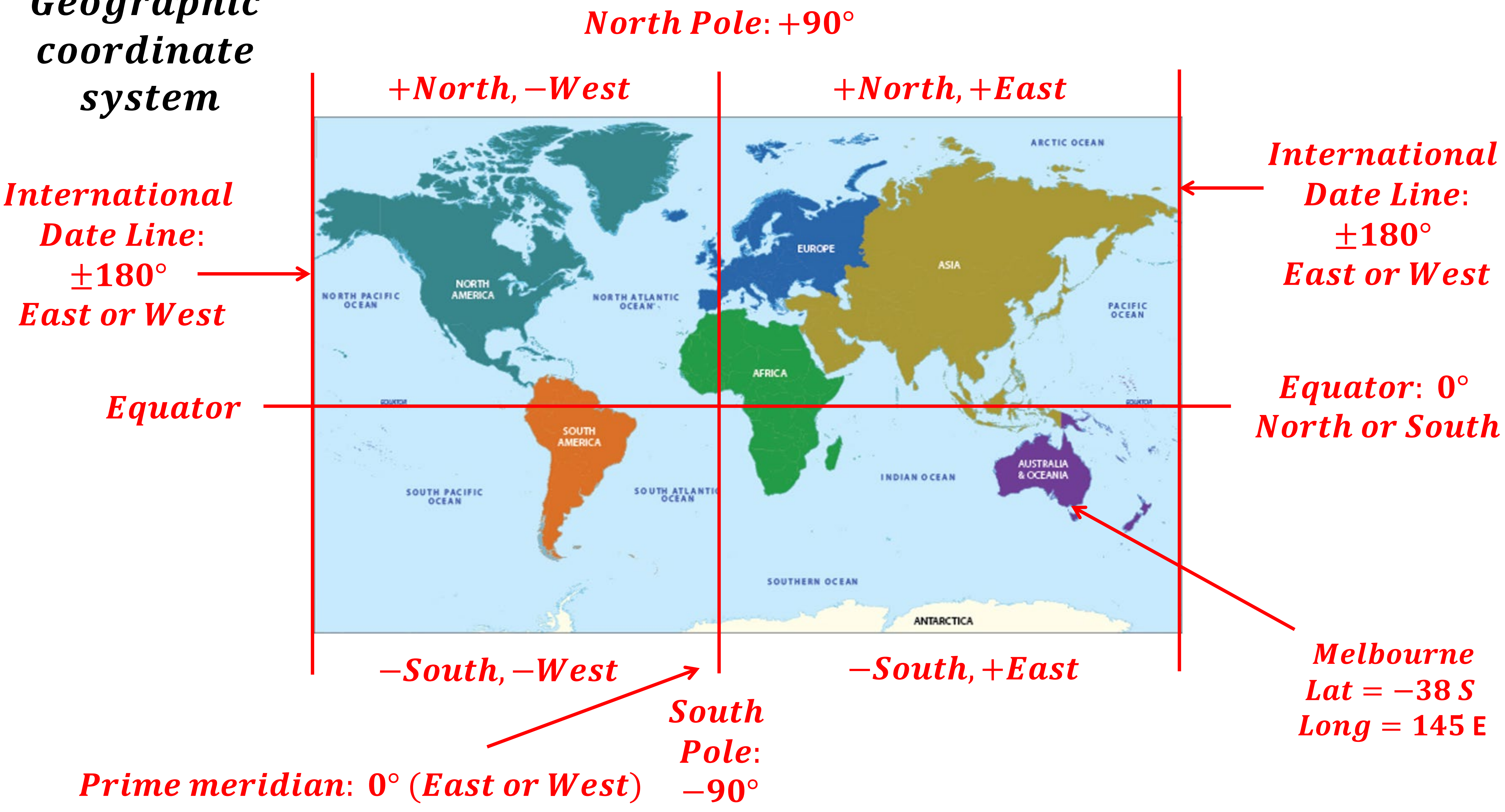
Figure 1. The radius of a circle of latitude is the Earth's radius R multiplied by the cosine of the latitude ϕ. This will apply to the circumference and all distances along this parallel of latitude.

This leads to the following calculations.

The theory



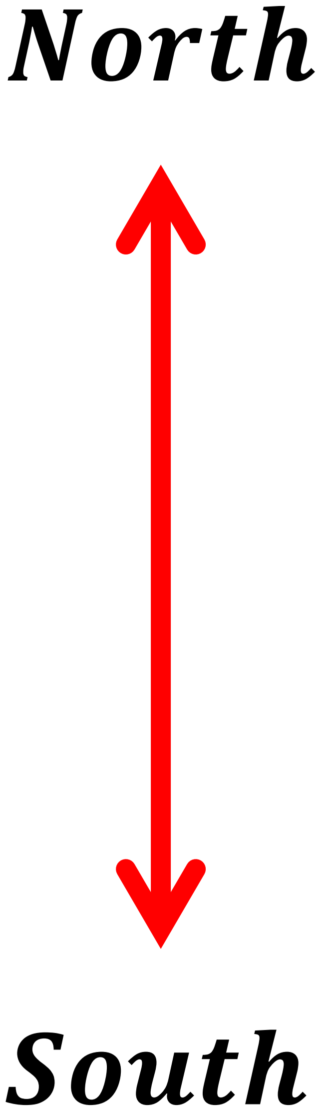
Geographic coordinate system

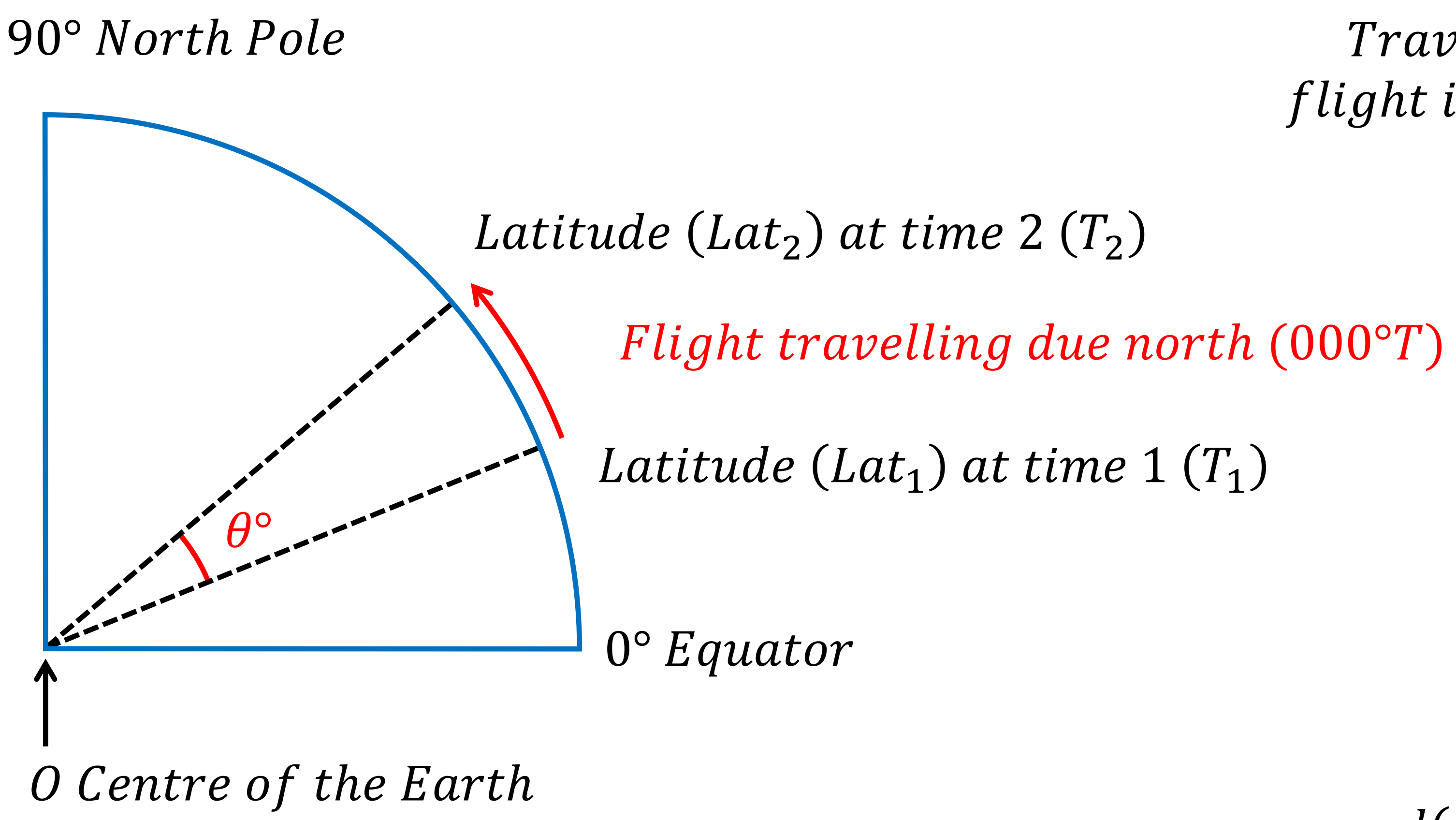


Method 1

Flight travelling due north ($000°T$) *or due south* ($180°T$)

(*No trigonometry*)





$\theta°$ is the difference in latitude from time 1 to time 2.

Travelling due north means the flight is on a great circle (meridian).

$$\theta° = |Lat_2 - Lat_1|$$

$$\Delta T(seconds) = video\ length$$

$$distance(nm) = \theta° \times 60$$

$$speed(knots) = \frac{distance(nm)}{\Delta T(seconds)} \times 3600$$

$$speed(km/h) = speed(knots) \times 1.852$$

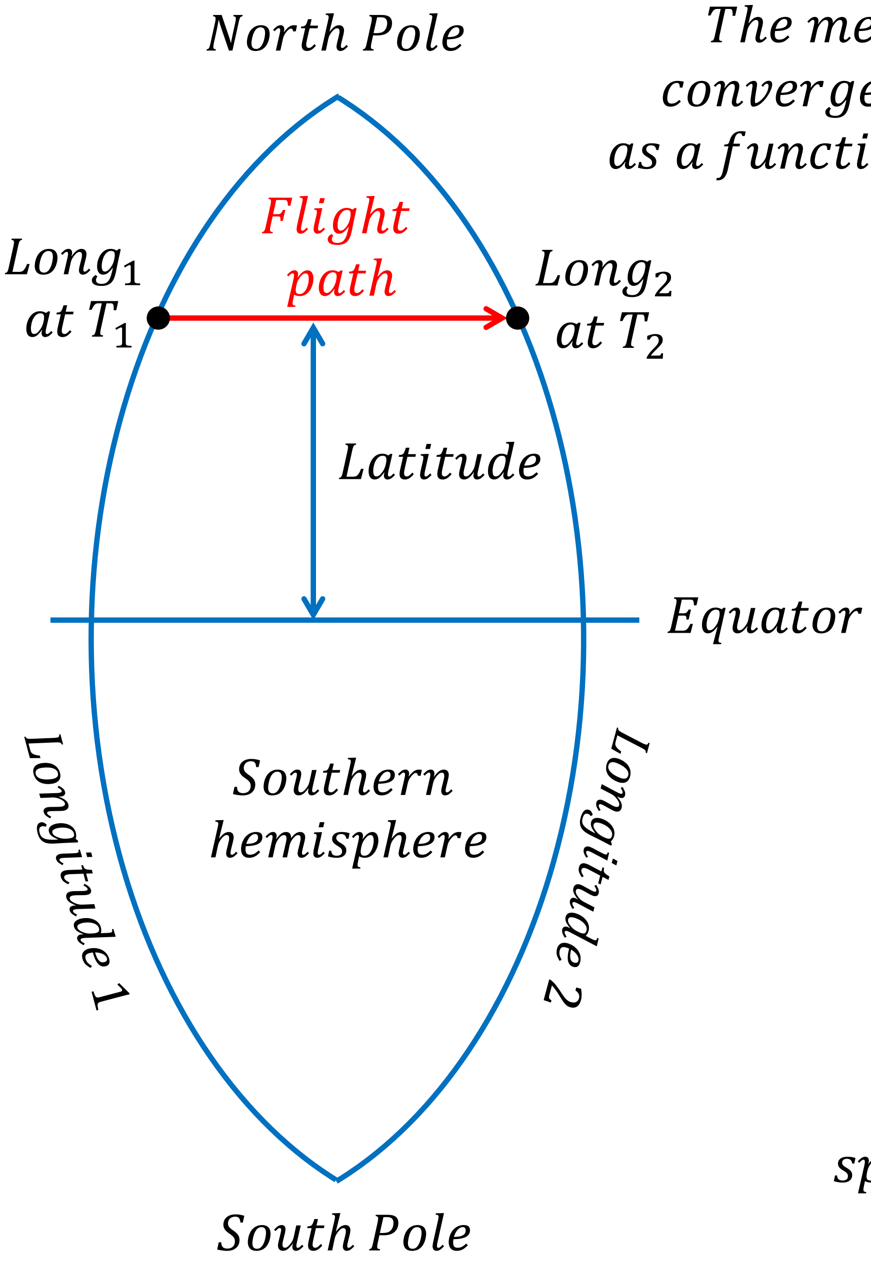
$$Track = 000°T$$

Method 2

Flight travelling
due east (090°T)
or due west (270°T)

West **East**

(Trigonometric correction)



The meridians (lines of longitude) converge at the North and South Poles as a function of the cosine of the latitude .

Travelling due east means the flight is on a parallel.

Flight travelling due east (090°T)

$$\Delta Long = |Long_2 - Long_1|$$

$$\Delta Time(seconds) = video\ length$$

$$distance(nm) = \Delta Long \times \cos(Latitude°) \times 60$$

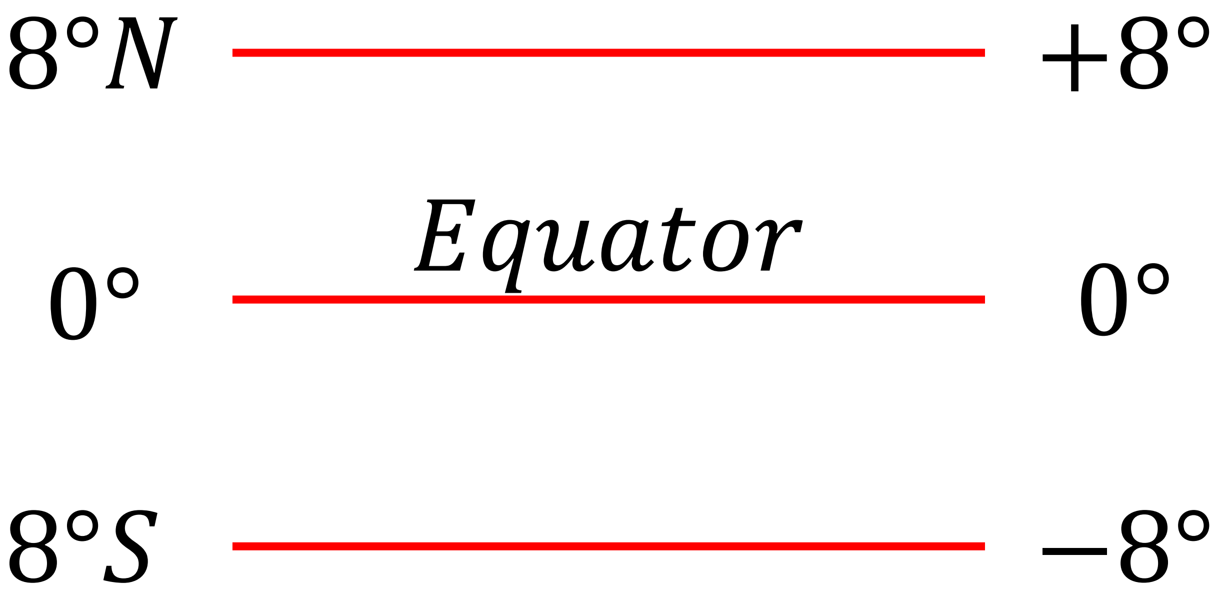
$$speed(knots) = \frac{distance(nm)}{\Delta T(seconds)} \times 3600$$

$$speed(km/h) = speed(knots) \times 1.852 \qquad Track = 090°T$$

Method 3

Flight travelling in any direction but within $\pm 8°$ *of the equator*

(Plane Trigonometry)

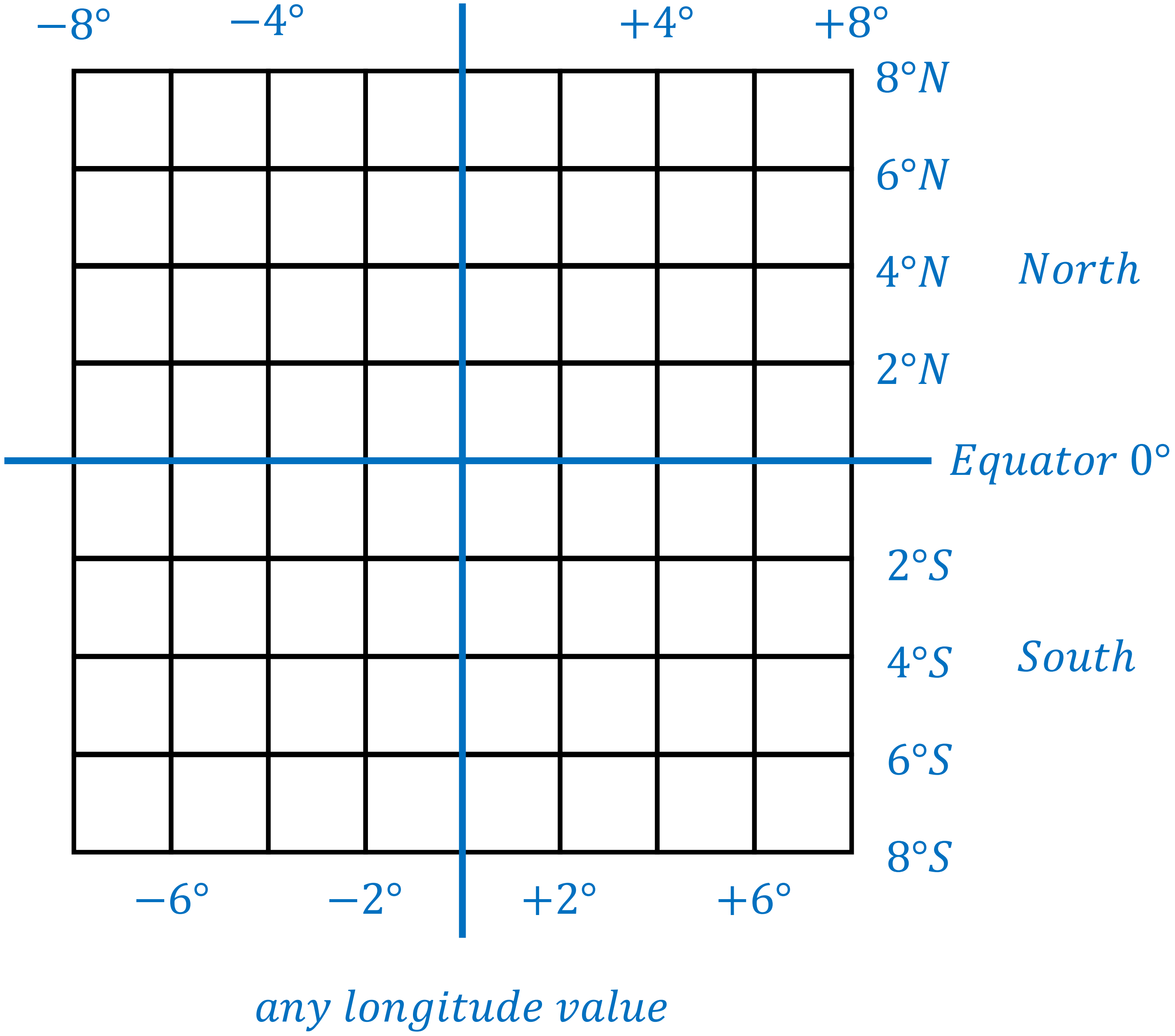


$$Why \pm 8° \ of \ the \ equator?$$

$$Because \ at \ \pm 8° \ the \ ratio \ Lat: Long = 100: 99$$

$$\cos(\pm 8°) = 0.990$$

$$This \ means \ that \ the \ latitude \ and \ longitude \ lines \ are \ close \ enough \ to \ squares \ and \ can \ be \ used \ as \ Cartesian \ coordinates \ where \ the \ units \ of \ x \ and \ y \ are \ in \ degrees \ of \ longitude \ and \ latitude \ respectively.$$



Within $\pm 8°$ *of the equator, the latitude and longitude lines form an almost perfect grid.*

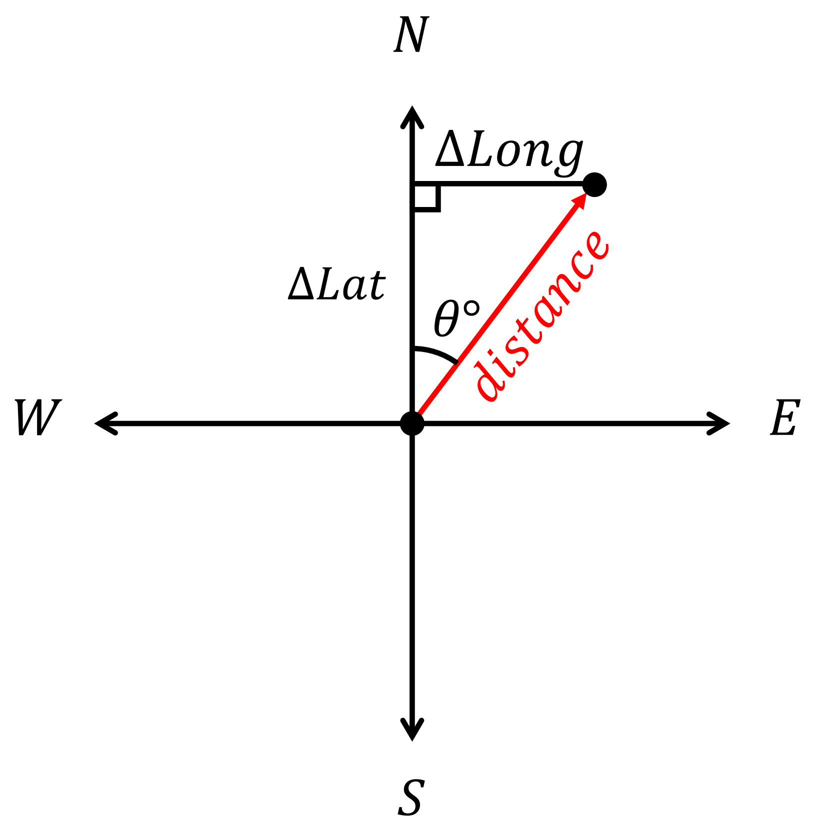
$Ratio\ of\ Lat\!:\!Long = 100\!:\!99$

$1\%\ error\ @\ \pm 8°\ Latitude$

$< 1\%\ error\ @\ < \pm 8°\ Latitude$

We can then use plane trigonometry to calculate distance, speed and track.

$NE\ quadrant\ 001°T \leq Track \leq\ 089°T$



$$1\ nautical\ mile\ (nm) = \frac{1}{60}\ degree$$

$$1\ knot = 1nm/h = 1.852\ km/h$$

$$\Delta Lat = |Latitude_2 - Latitude_1|$$

$$\Delta Long = |Longitude_2 - Longitude_1|$$

$$\Delta T(seconds) = video\ length$$

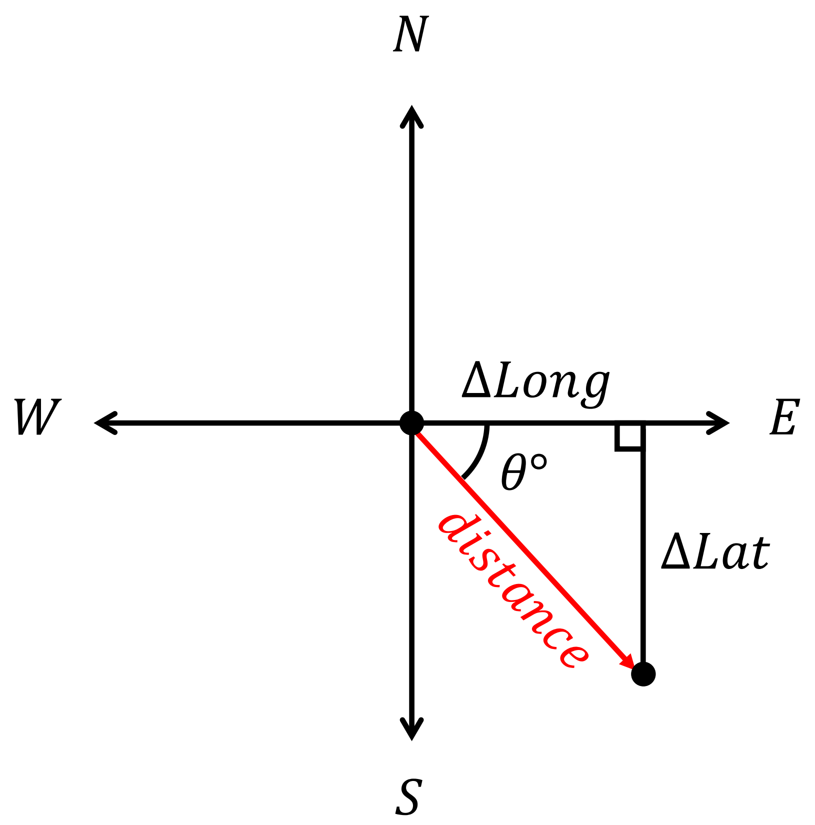
$$distance(\theta°) = \sqrt{(\Delta Lat)^2 + (\Delta Long)^2}$$

$$distance(nm) = distance\ (\theta°) \times 60$$

$$speed(knots) = \frac{distance(nm)}{\Delta T(seconds)} \times 3600$$

$$track = bearing = 000° + \tan^{-1}\left(\frac{\Delta Long}{\Delta Lat}\right)$$

$SE\ quadrant\ 091°T \leq Track \leq 179°T$



$$1\ nautical\ mile\ (nm) = \frac{1}{60}\ degree$$

$$1\ knot = 1nm/h = 1.852\ km/h$$

$$\Delta Lat = |Latitude_2 - Latitude_1|$$

$$\Delta Long = |Longitude_2 - Longitude_1|$$

$$\Delta T(seconds) = video\ length$$

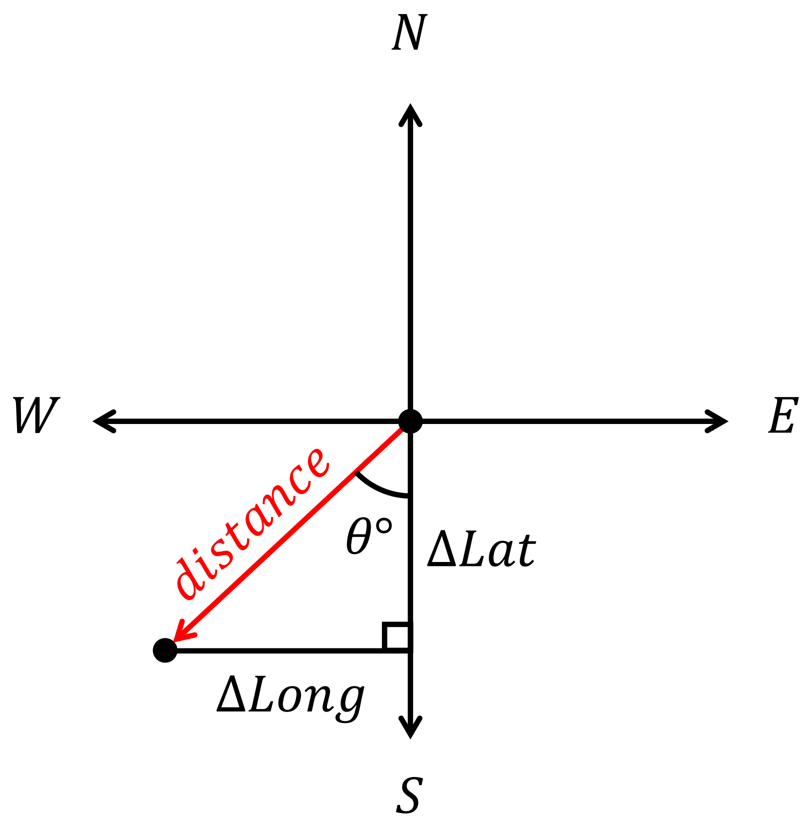
$$distance(\theta°) = \sqrt{(\Delta Lat)^2 + (\Delta Long)^2}$$

$$distance(nm) = distance\ (\theta°) \times 60$$

$$speed(knots) = \frac{distance(nm)}{\Delta T(seconds)} \times 3600$$

$$track = bearing = 090° + \tan^{-1}\left(\frac{\Delta Lat}{\Delta Long}\right)$$

$SW\ quadrant\ 181°T \leq Track \leq 269°T$



$$\Delta Lat = |Latitude_2 - Latitude_1|$$

$$\Delta Long = |Longitude_2 - Longitude_1|$$

$$\Delta T(seconds) = video\ length$$

$$distance(\theta°) = \sqrt{(\Delta Lat)^2 + (\Delta Long)^2}$$

$$distance(nm) = distance\ (\theta°) \times 60$$

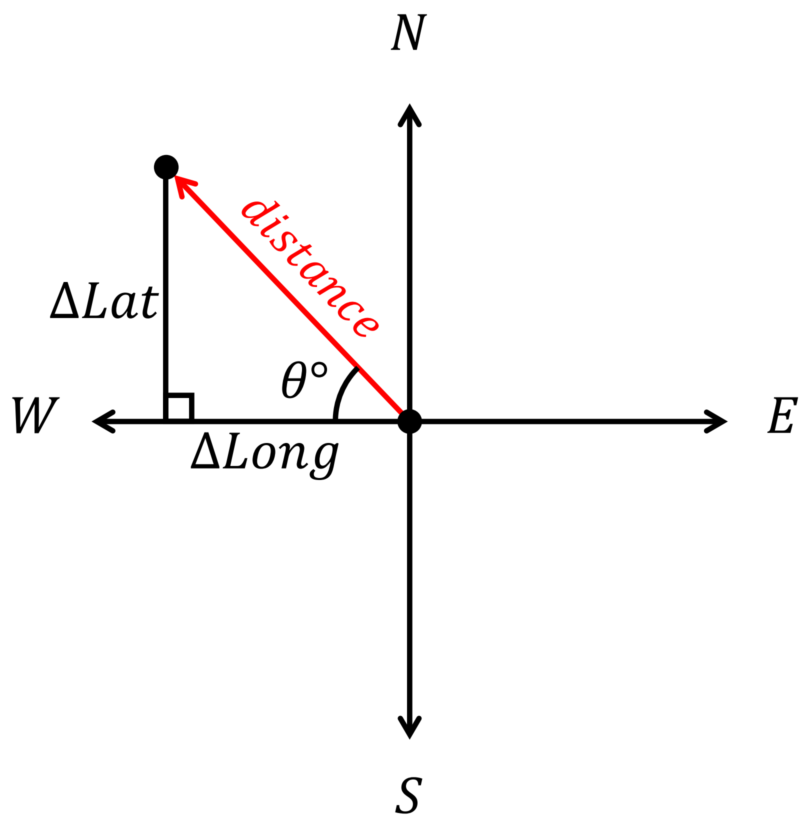
$$speed(knots) = \frac{distance(nm)}{\Delta T(seconds)} \times 3600$$

$$track = bearing = 180° + \tan^{-1}\left(\frac{\Delta Long}{\Delta Lat}\right)$$

$$1\ nautical\ mile\ (nm) = \frac{1}{60}\ degree$$

$$1\ knot = 1nm/h = 1.852\ km/h$$

$NW\ quadrant\ 271°T \leq Track \leq 359°T$



$$\Delta Lat = |Latitude_2 - Latitude_1|$$

$$\Delta Long = |Longitude_2 - Longitude_1|$$

$$\Delta T(seconds) = video\ length$$

$$distance(\theta°) = \sqrt{(\Delta Lat)^2 + (\Delta Long)^2}$$

$$distance(nm) = distance\ (\theta°) \times 60$$

$$speed(knots) = \frac{distance(nm)}{\Delta T(seconds)} \times 3600$$

$$track = bearing = 270° + \tan^{-1}\left(\frac{\Delta Lat}{\Delta Long}\right)$$

$$1\ nautical\ mile\ (nm) = \frac{1}{60}\ degree$$

$$1\ knot = 1nm/h = 1.852\ km/h$$

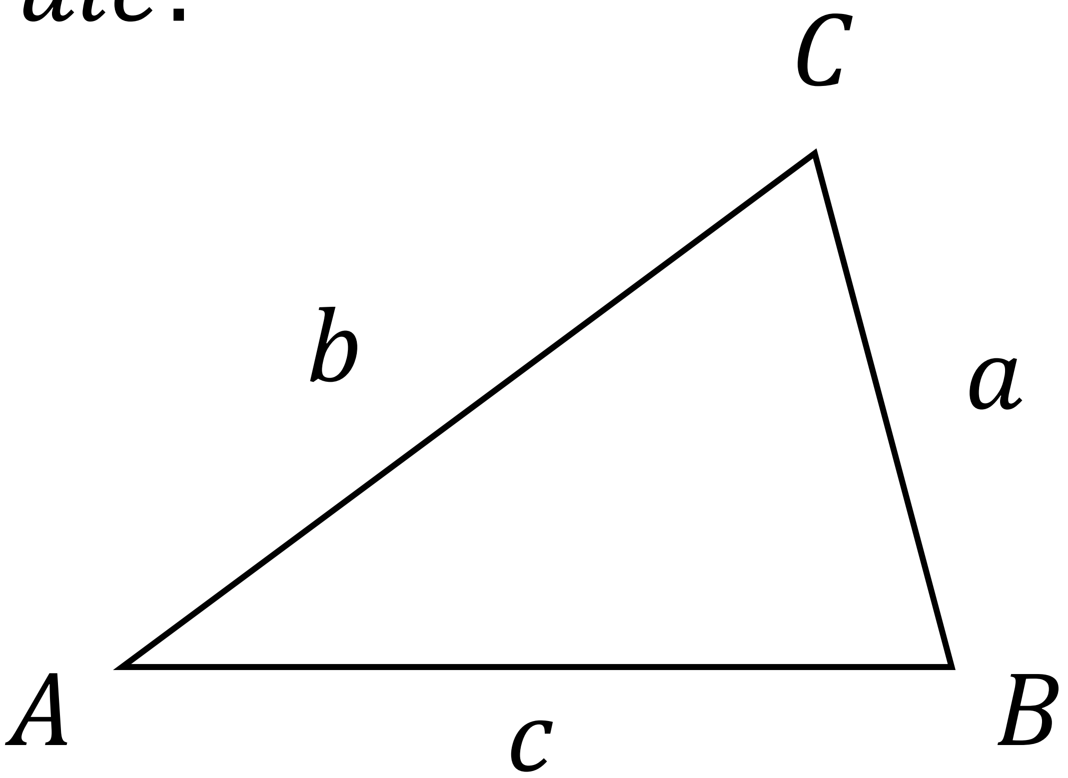
Method 4

Flight travelling in
any *direction and*
anywhere *on the Earth*

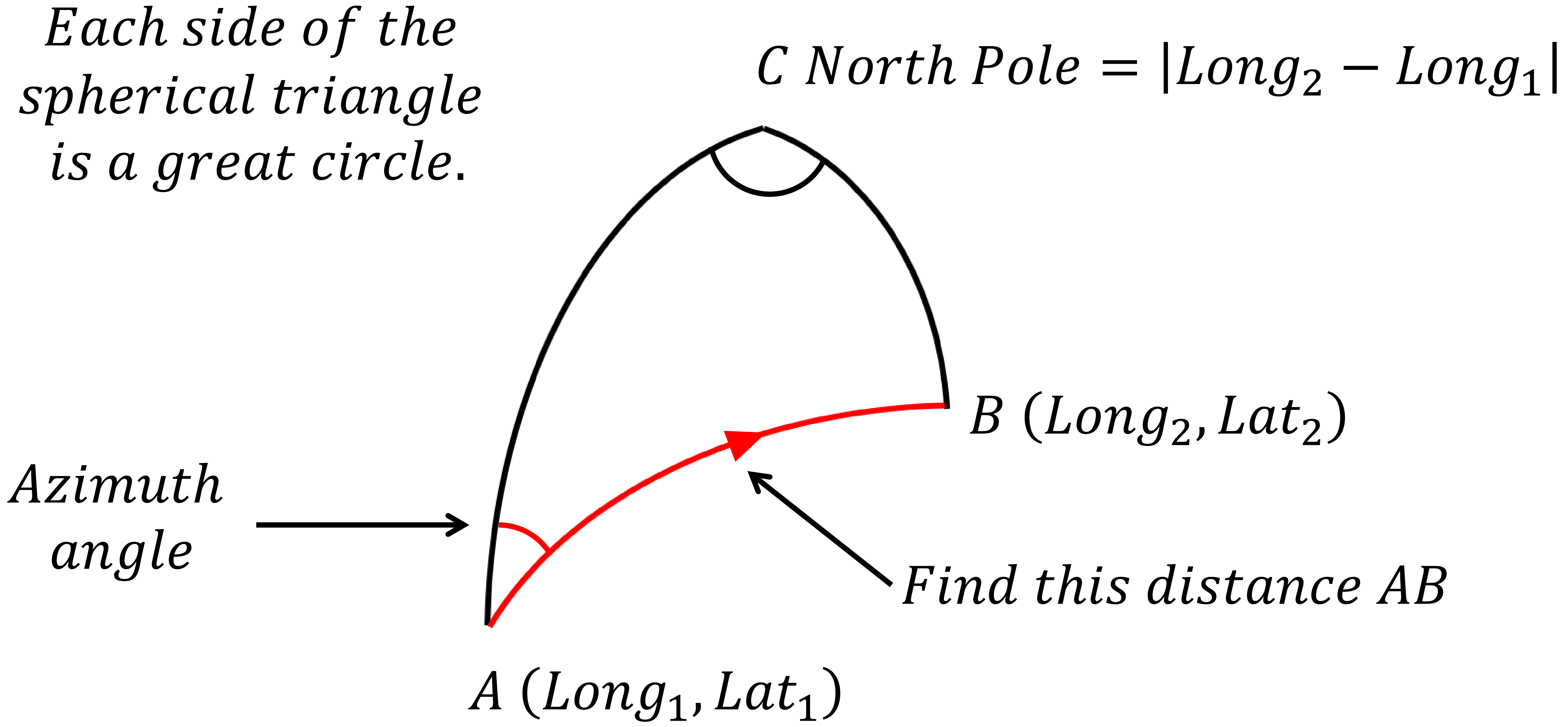
Based on spherical trigonometry

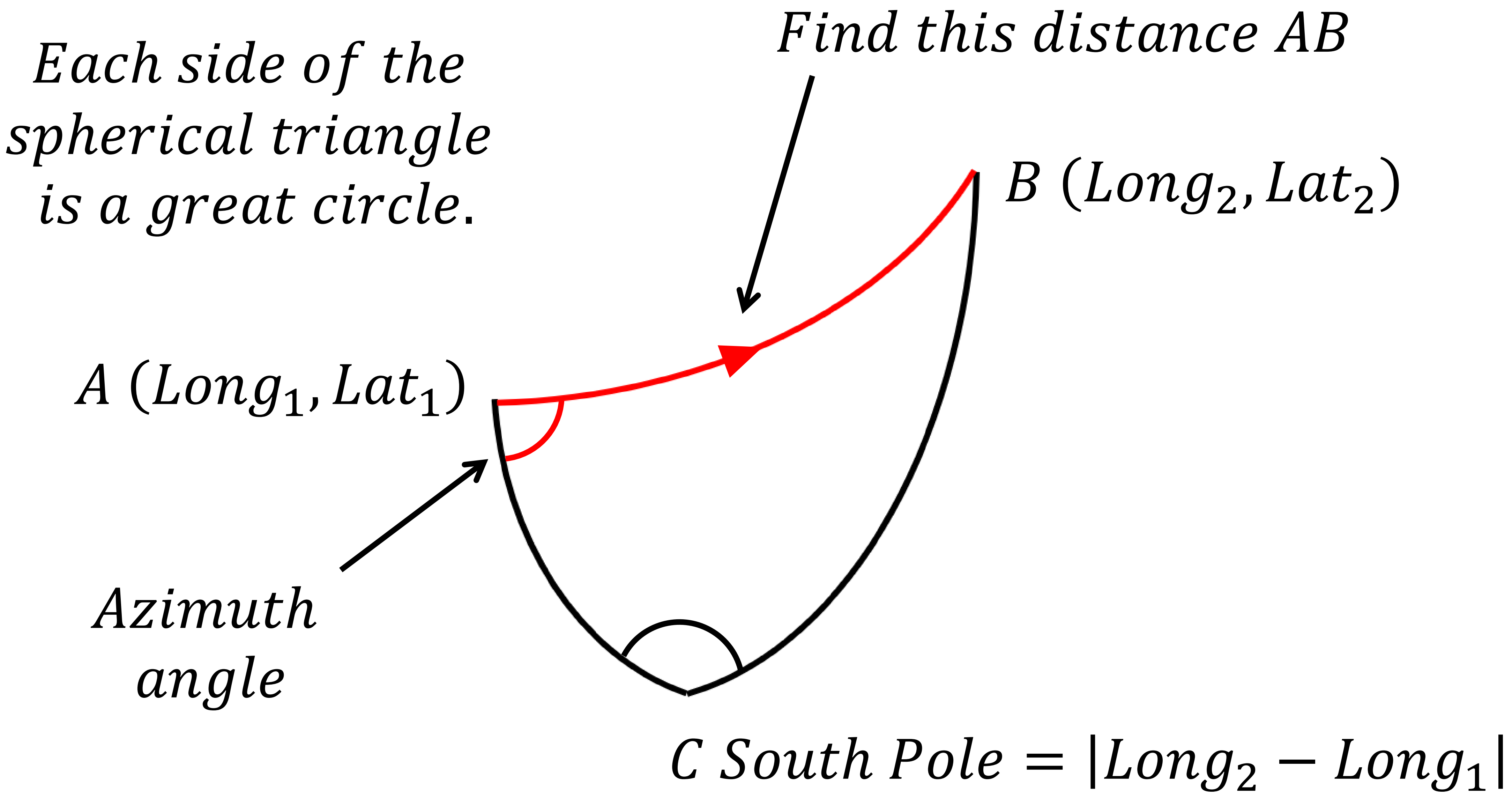
Uses the great circle distance formula

The cosine rule:



$$c^2 = a^2 + b^2 - 2ab\cos(C)$$


Each side of the spherical triangle is a great circle.
C North Pole $= |Long_2 - Long_1|$
$B\ (Long_2, Lat_2)$
Azimuth angle
Find this distance AB
$A\ (Long_1, Lat_1)$



Each side of the spherical triangle is a great circle.
Find this distance AB
$B\ (Long_2, Lat_2)$
$A\ (Long_1, Lat_1)$
Azimuth angle
C South Pole $= |Long_2 - Long_1|$

$\boldsymbol{Part\ 1}$: $\boldsymbol{Calculating\ the\ speed\ in\ knots}$ ($\boldsymbol{\&\ km/h}$)

$Using\ the\ great\ circle\ distance\ formula.\quad The\ spherical\ version\ of\ the\ cosine\ rule.$

$$arc\ angle = \cos^{-1}[\sin(Lat_1)\sin(Lat_2) + \cos(Lat_1)\cos(Lat_2)\cos(|Long_2 - Long_1|)]$$

$$distance(nm) = arc\ angle \times 60$$

$nm = nautical\ miles$ $\qquad 1\ nm = 1.852\ km$ $\qquad knots = nm/h$

$$speed(knots) = \frac{distance(nm)}{\Delta T(seconds)} \times 3600$$

$$speed(km/h) = speed(knots) \times 1.852$$

$\boldsymbol{Part\ 1}$: $\boldsymbol{Calculating\ the\ speed\ in\ knots}$ $(\&\ \boldsymbol{km/h})$

$Using\ the\ great\ circle\ distance\ formula.$ $The\ spherical\ version\ of\ the\ cosine\ rule.$

$$AB = 60 \times \cos^{-1}[\sin(Lat_1)\sin(Lat_2) + \cos(Lat_1)\cos(Lat_2)\cos(|Long_2 - Long_1|)]$$

$$AB = distance(nm)$$

$nm = nautical\ miles$ $\quad 1\ nm = 1.852\ km$ $\quad knots = nm/h$

$$speed(knots) = \frac{distance(nm)}{\Delta T(seconds)} \times 3600$$

$$speed(km/h) = speed(knots) \times 1.852$$

$\boldsymbol{Part\ 2: Calculating\ the\ Track\ (Bearing)}$

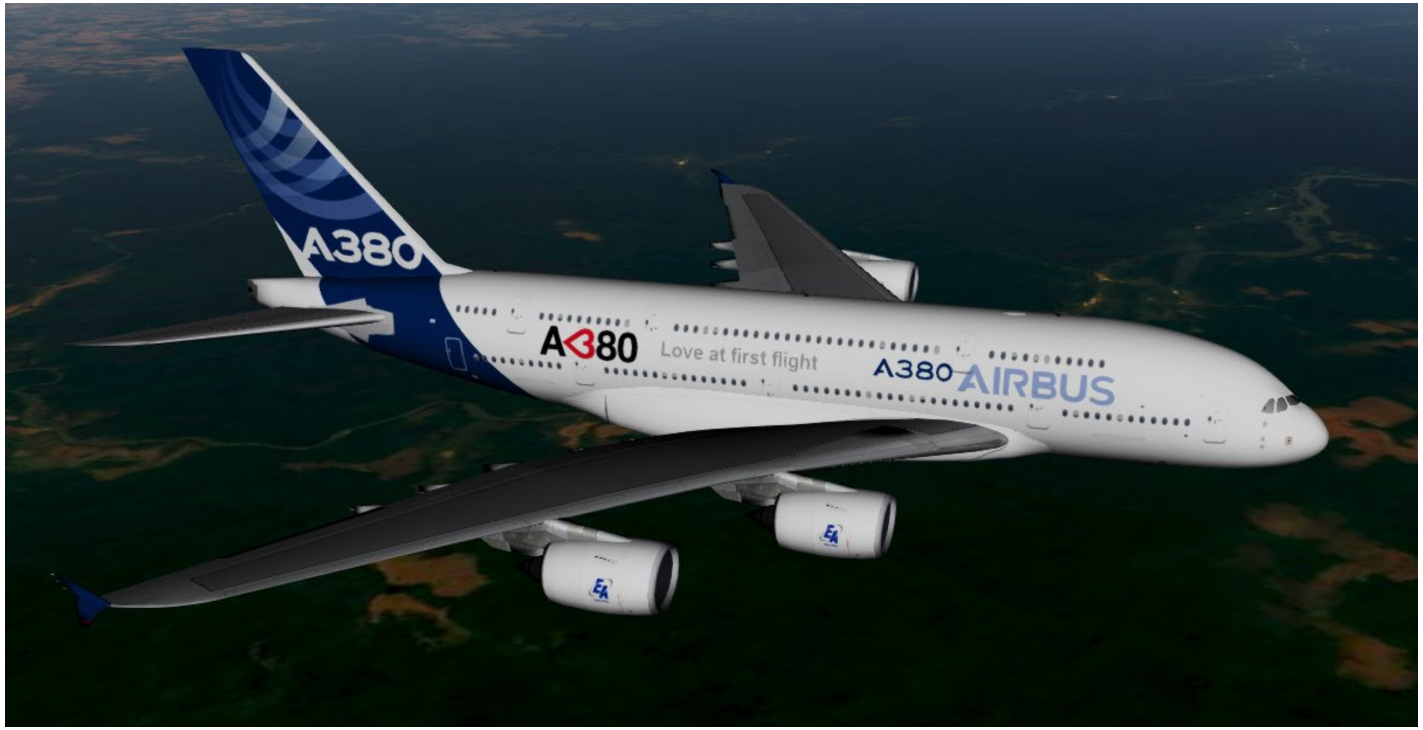
$$Azimuth\ angle: \quad \theta° = \cos^{-1}\left(\frac{\sin(Lat_2) - \sin(Lat_1)\cos(arc\ angle)}{\cos(Lat_1)\sin(arc\ angle)}\right)$$

$Then\ apply\ the\ following\ correction$
$to\ have\ the\ Track\ from\ 000°T\ to\ 359°T$

$Lon_2\ EAST\ of\ Lon_1 \quad i.e.\ if\ Lon_2 > Lon_1\ then\ Track = \theta°$

$Lon_2\ WEST\ of\ Lon_1 \quad i.e.\ if\ Lon_2 < Lon_1\ then\ Track = 360° - \theta°$

The practice



For the puposes of this presentation,
all the calculations will be done using
the ***TI CAS calculator*** *as this will show all*
the steps to get to the flight's speed and track.
Make sure the calculator is set to ***DEGREES*** *mode.*

To do these calculations repeatedly,
it is best to use a spreadsheet such as
MICROSOFT EXCEL *or* ***GOOGLE SHEETS***
to automate these calculations.

Note:
Spreadsheets use
radians as the default
unit of angular measurement.

To convert between radians and degrees,
use the RADIANS and DEGREES functions.

Examples:

=DEGREES(PI()) → 180 degrees

=RADIANS(45) → 0.7854 radians

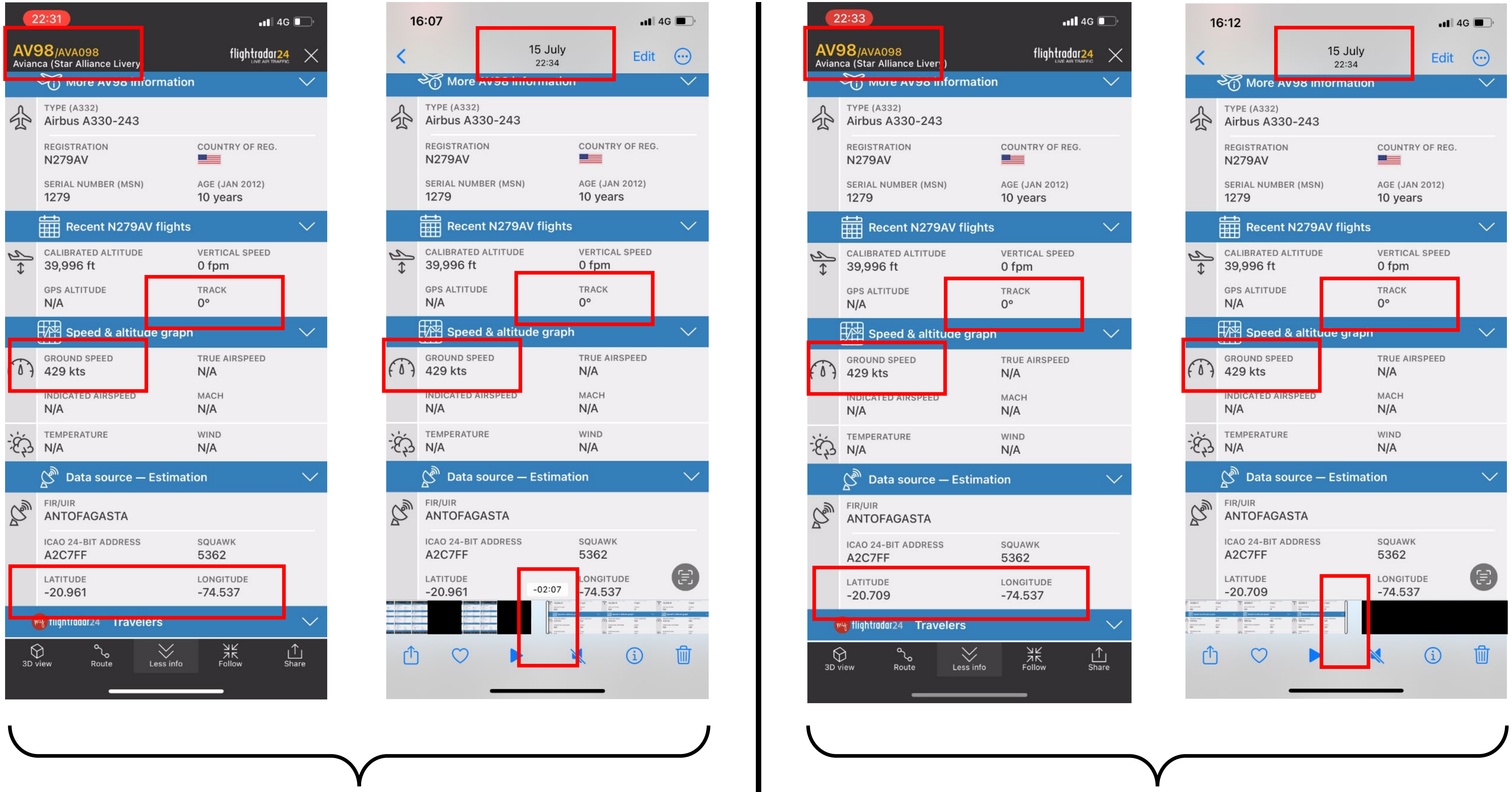
2022 Flightradar24 Data Set used in the following examples

No.	Date (2022)	Flight No.	From	To	Lat 1 (decimal)	Long 1 (decimal)	Lat 2 (decimal)	Long 2 (decimal)	Δ Time (mm:ss)	Speed (kts)	Track (° True)
1	15 Jul	AV98	SCL	BOG	-20.961	-74.537	-20.709	-74.537	02:07	429	0
2	22 Jul	LA800	AKL	SCL	-57.109	-135.995	-57.109	-135.375	02:32	477	90
3	25 Jun	AF229	EZE	CDG	6.791	-23.793	7.157	-23.636	02:59	481	023
4	28 Jun	QF63	SYD	JNB	-48.286	68.475	-48.179	68.055	03:04	440	291

Input data *Objective*

$\boldsymbol{Method\ 1}$

$Flight\ travelling$
$due\ north\ (000°T)$
$or\ due\ south\ (180°T)$



Data at the ***START*** *of recording*

Data at the ***END*** *of recording*

Method 1: Flight travelling north

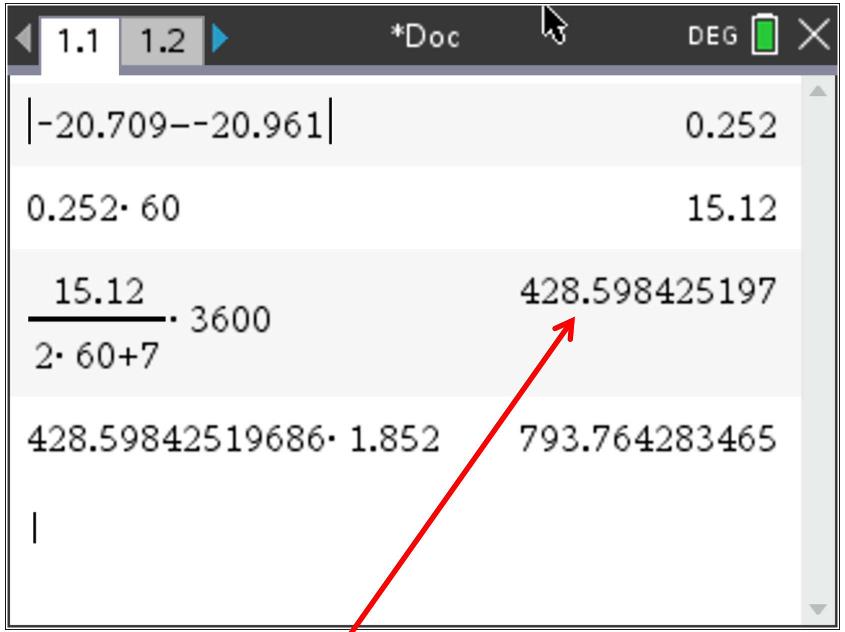
Date	2022-07-15
Flight Number	AV98
From	Santiago SCL
To	Bogota BOG
Latitude1 (degrees)	-20.961
Longitude1 (degrees)	-74.537
Latitude2 (degrees)	-20.709
Longitude2 (degrees)	-74.537
ΔTime (mm:ss)	02:07
Speed (knots)	429
Track (°T)	000

Angle°

Distance in nm

Speed in knots

Speed in km/h



$Speed \approx 429\ knots$

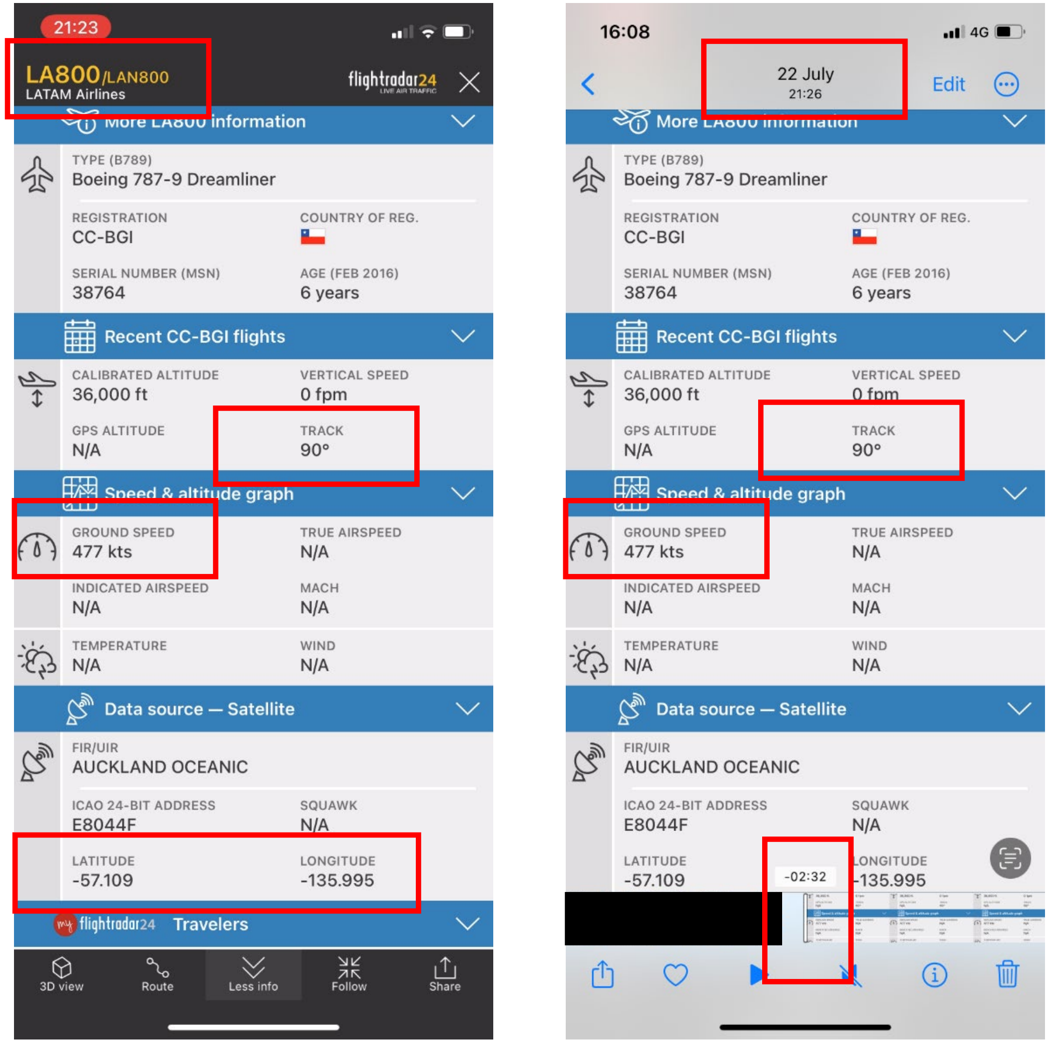
$Track = 000°\ True \because Long_1 = Long_2\ and\ Lat_2\ North\ of\ Lat_1$

Flight Calculations Travelling due NORTH								✗	✓
4	Edit only the red cells		Calculations in BLUE		Δ Latitude	Δ Longitude		Satellite	4G
Date	2022-07-15				0.252	0		✗	✓
Flight No.	AV98								
From	Santiago	SCL		C = 40 000 km	r = 6371 km	r + altitude			
To	Bogota	BOG							
			Long dist (km)	28.0	28.0	28.1			
Altitude (feet)	39996								
Altitude (m)	12191								
ΔT ('mmss)	0207		Speed (km/h)	794	794	796			
ΔT (s)	127		Speed (knots)	429	429	430			
Speed (knots)	429								
Speed (km/h)	795		Speed error (knots)	0	0	1			
			Speed error (%)	-0.10%	-0.03%	0.17%			
Track (° True)	0								
T1 Latitude	-20.961		atan() (decimal degrees)	0.0	0.0	0.0			
T1 Longitude	-74.537								
T2 Latitude	-20.709								
T2 Longitude	-74.537			Bearing					
				Track					
				0 < θ < 90	Track				
				(dec degs)	error				
				0.00	0.00				

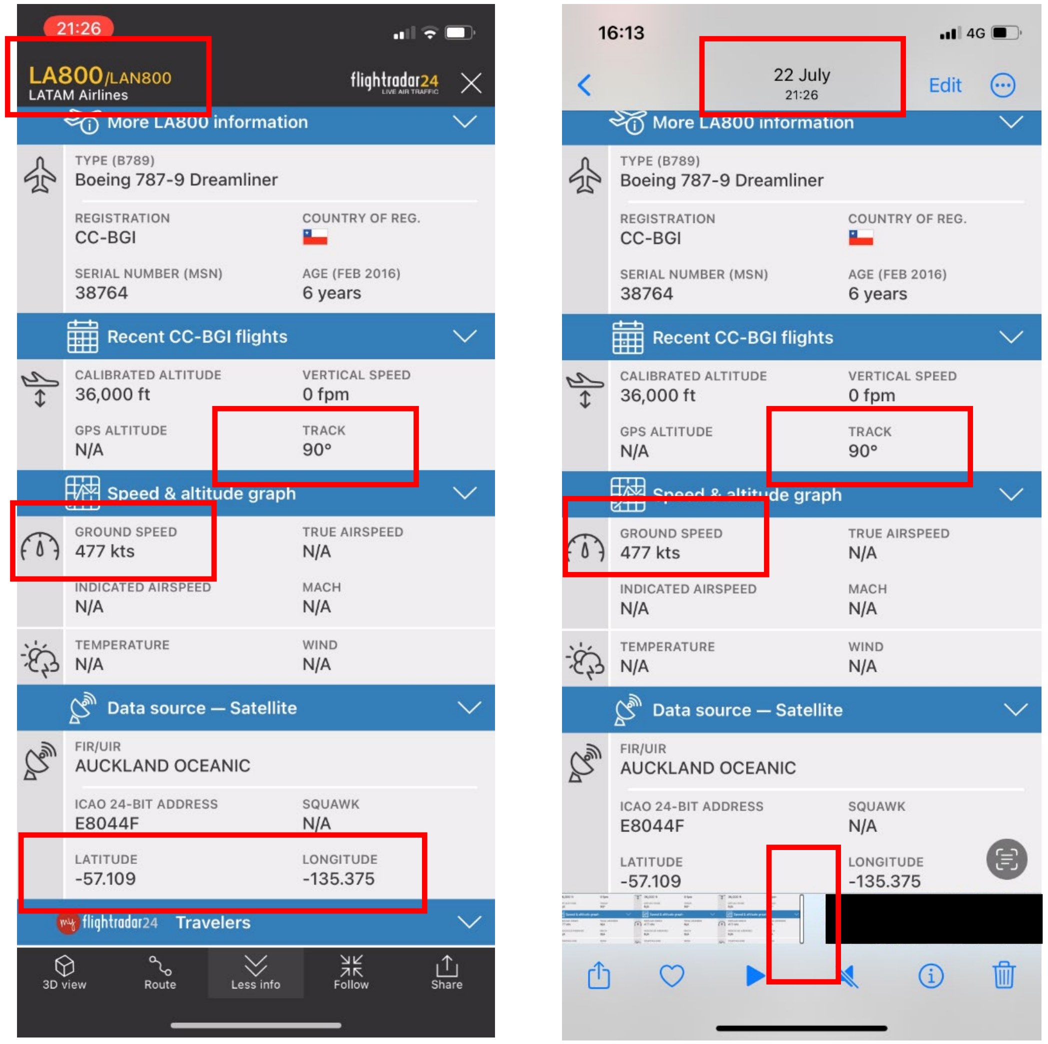
$$Track = 0^\circ + \tan^{-1}\left(\frac{\Delta Longitude}{\Delta Latitude}\right)$$

$\boldsymbol{Method\ 2}$

$Flight\ travelling$
$due\ east (090°T)$
$or\ due\ west\ (270°T)$



Data at the ***START*** of recording



Data at the ***END*** of recording

$Method\ 2$: $Flight\ travelling\ east$

Date	2022-07-22
Flight Number	LA800
From	Auckland AKL
To	Santiago SCL
Latitude1 (degrees)	-57.109
Longitude1 (degrees)	-135.995
Latitude2 (degrees)	-57.109
Longitude2 (degrees)	-135.375
ΔTime (mm:ss)	02:32
Speed (knots)	477
Track (˚T)	090

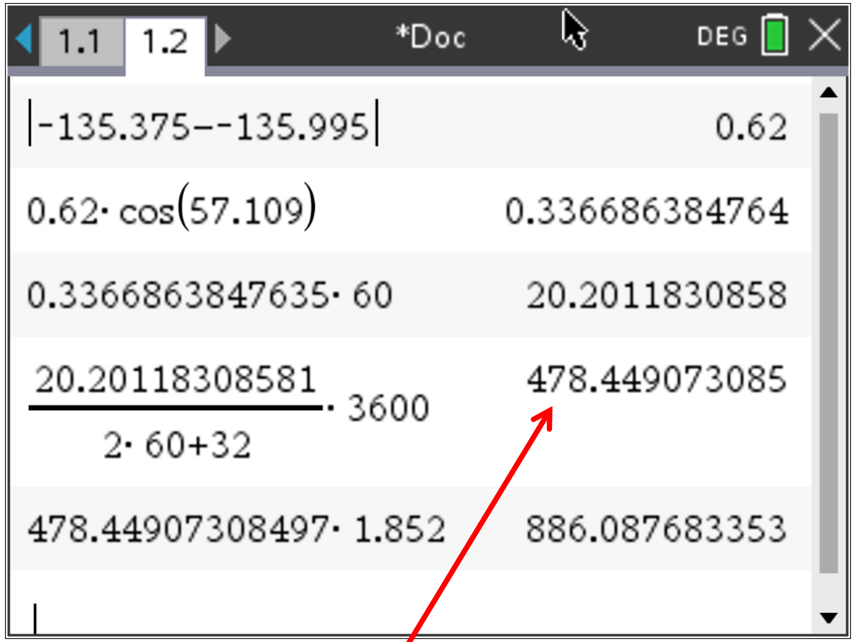
$Angle°$

$Angle \times \cos(Lat°)$

$Distance\ in\ nm$

$Speed\ in\ knots$

$Speed\ in\ km/h$



$Speed\ \approx 477\ knots$

$Track\ = 090°\ True\ \because Lat_1 = Lat_2\ and\ Long_2\ East\ of\ Long_1$

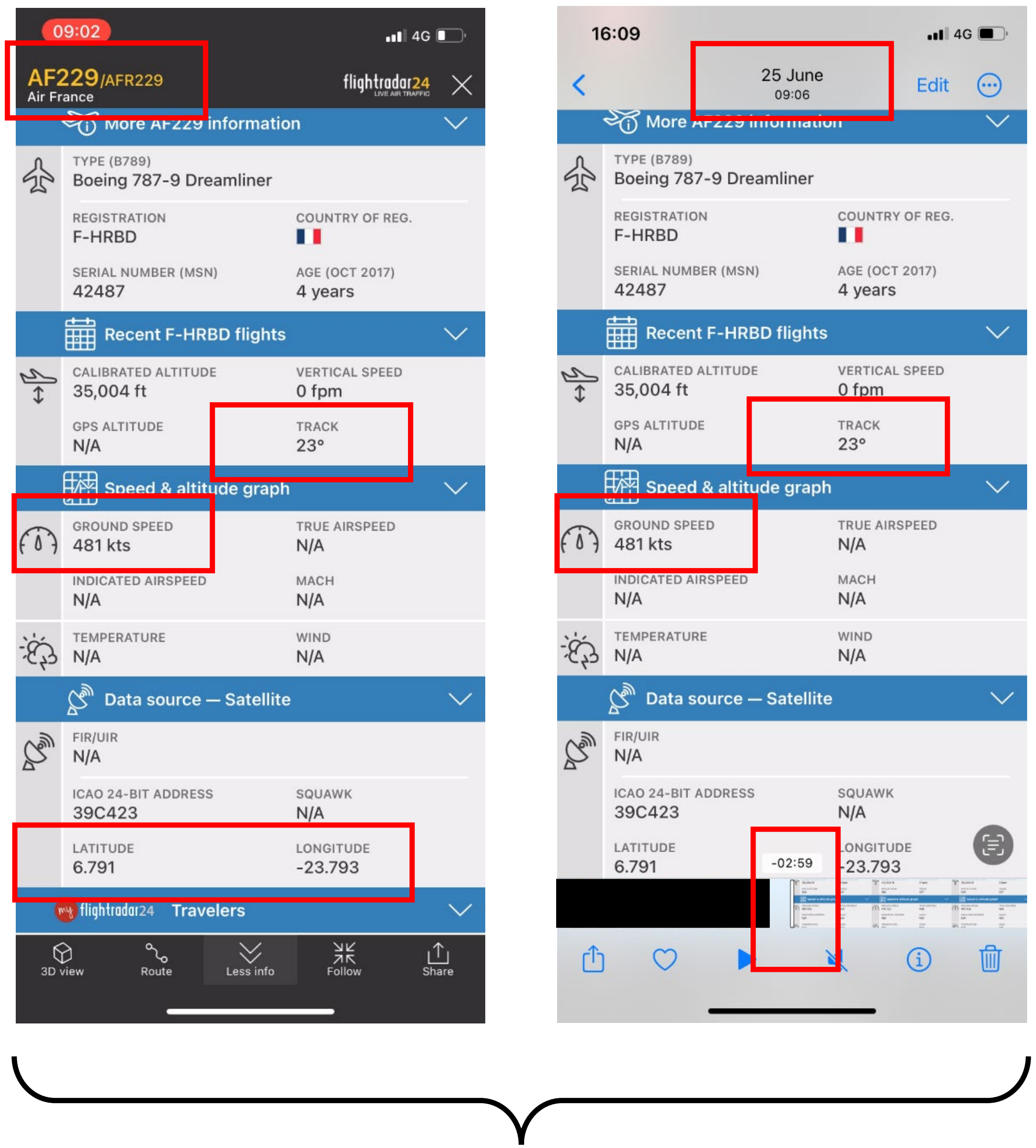
Flight Calculations Travelling due EAST								✗	✓
5	Edit only the red cells		Calculations in BLUE		Δ Latitude	Δ Longitude		Satellite	4G
Date	2022-07-22				0	0.62		✓	✗
Flight No.	LA800								
From	Auckland	AKL		C = 40 000 km	r = 6371 km	r + altitude			
To	Santiago	SCL							
			Long dist (km)	37.4	37.4	37.5			
Altitude (feet)	36000								
Altitude (m)	10973								
ΔT ('mmss)	0232		Speed (km/h)	886	887	888			
ΔT (s)	152		Speed (knots)	478	479	480			
Speed (knots)	477								
Speed (km/h)	883		Speed error (knots)	1	2	3			
			Speed error (%)	0.30%	0.37%	0.54%			
Track (° True)	90								
T1 Latitude	-57.109		atan() (decimal degrees)	90.0	90.0	90.0			
T1 Longitude	-135.995								
T2 Latitude	-57.109								
T2 Longitude	-135.375			Bearing					
				Track					
				0 < θ < 90	Track				
				(dec degs)	error				
				90.00	0.00				

$$Track = 90^{\circ} + \tan^{-1}\left(\frac{\Delta Latitude}{\Delta Longitude}\right)$$

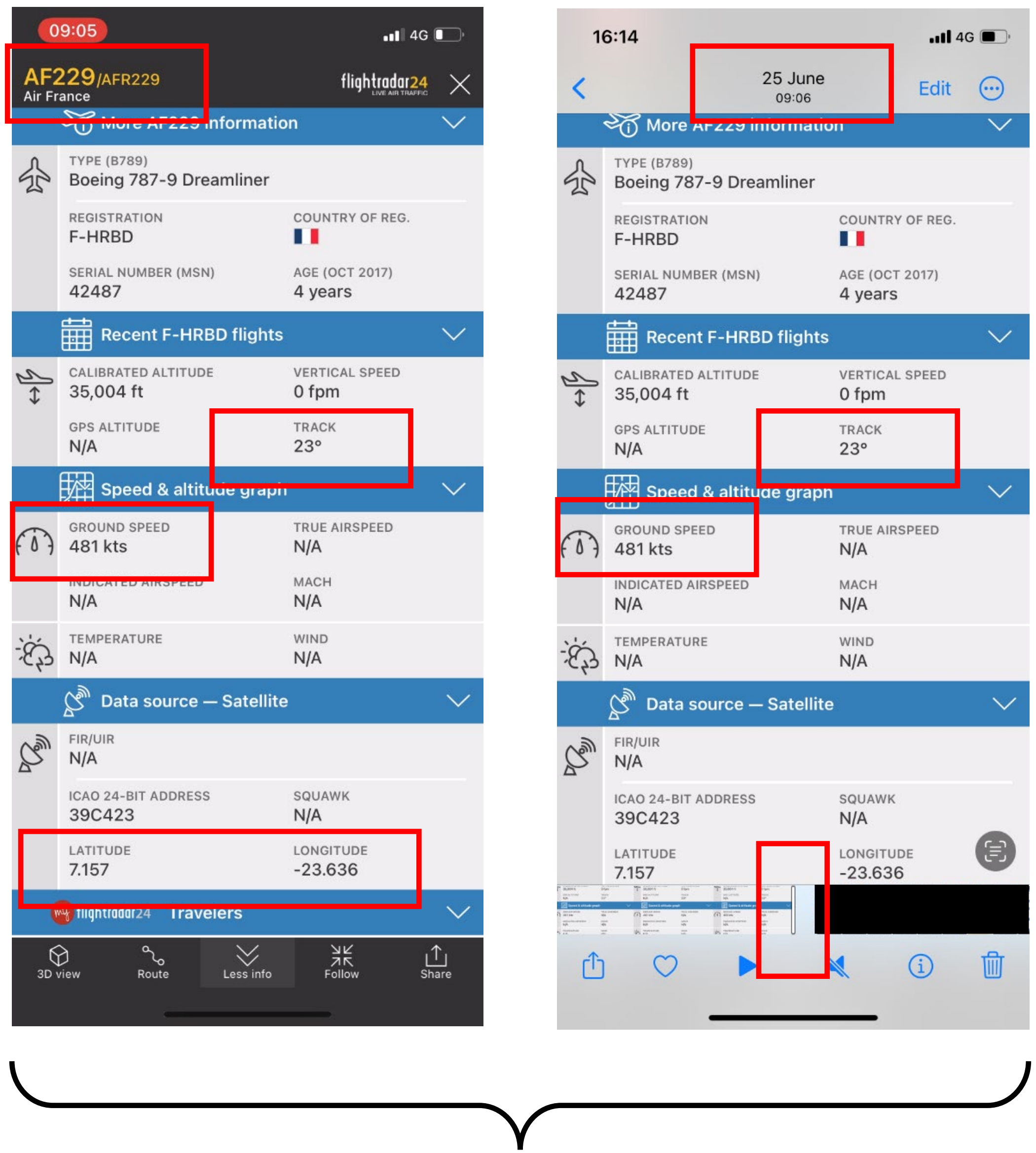
Method 3

Flight travelling in any direction but within $\pm 8°$ *of the equator*

Plane Trigonometry



*Data at the **START** of recording*



*Data at the **END** of recording*

Method 3: Flight near the equator

Date	2022-06-25
Flight Number	AF229
From	Buenos Aries EZE
To	Paris CDG
Latitude1 (degrees)	6.791
Longitude1 (degrees)	-23.793
Latitude2 (degrees)	7.157
Longitude2 (degrees)	-23.636
ΔTime (mm:ss)	02:59
Speed (knots)	481
Track (˚T)	023

1.1 1.2 1.3 *Doc DEG

$\Delta Latitude$	$\lvert 7.157-6.791 \rvert$	0.366
$\Delta Longitude$	$\lvert -23.636--23.793 \rvert$	0.157
$Track \; = 023° \; True$	$\tan^{-1}\left(\frac{0.157}{0.366}\right)$	23.2174813074
$Distance \; as \; an \; angle$	$\sqrt{(0.366)^2+(0.157)^2}$	0.39825243251
$Distance(nm)$	$0.39825243250983 \cdot 60$	23.8951459506

1.2 1.3 1.4 *Doc DEG

$Speed(knots)$	$\frac{23.89514595059}{2 \cdot 60+59} \cdot 3600$	480.572767721
$Speed(km/h)$	$480.57276772138 \cdot 1.852$	890.02076582

$Speed \; \approx 481 \; knots$

Flight Calculations near the equator			*Do NOT use a flight that crosses the equator at the International Date Line (IDL)*						✓	✗
6	Edit only the red cells		Calculations in BLUE		Δ Latitude	Δ Longitude		Satellite	4G	
								✓	✓	
Date	2022-06-25				0.366	0.157				
Flight No.	AF229									
From	Buenos Aires	EZE		C = 40 000 km	r = 6371 km	r + altitude				
To	Paris	CDG	Lat dist (km)	40.7	40.7	40.8				
			Long dist (km)	17.4	17.5	17.5				
Altitude (feet)	35004		Diag dist (km)	44.3	44.3	44.4				
Altitude (m)	10669									
ΔT ('mmss)	0259		Speed (km/h)	890	891	892				
ΔT (s)	179		Speed (knots)	481	481	482				
Speed (knots)	481									
Speed (km/h)	891		Speed error (knots)	0	0	1				
			Speed error (%)	-0.05%	-0.01%	0.08%				
Track (° True)	23									
			Δlong/ΔLat	0.4290	0.4290	0.4290				
T1 Latitude	6.791		LongDist/DiagDist	0.3942	0.3942	0.3942				
T1 Longitude	-23.793		LatDist/DiagDist	0.9190	0.9190	0.9190				
T2 Latitude	7.157		LatDist/LongDist	2.3312	2.3312	2.3312				
T2 Longitude	-23.636									
			atan() (decimal degrees)	23.2	23.2	23.2				

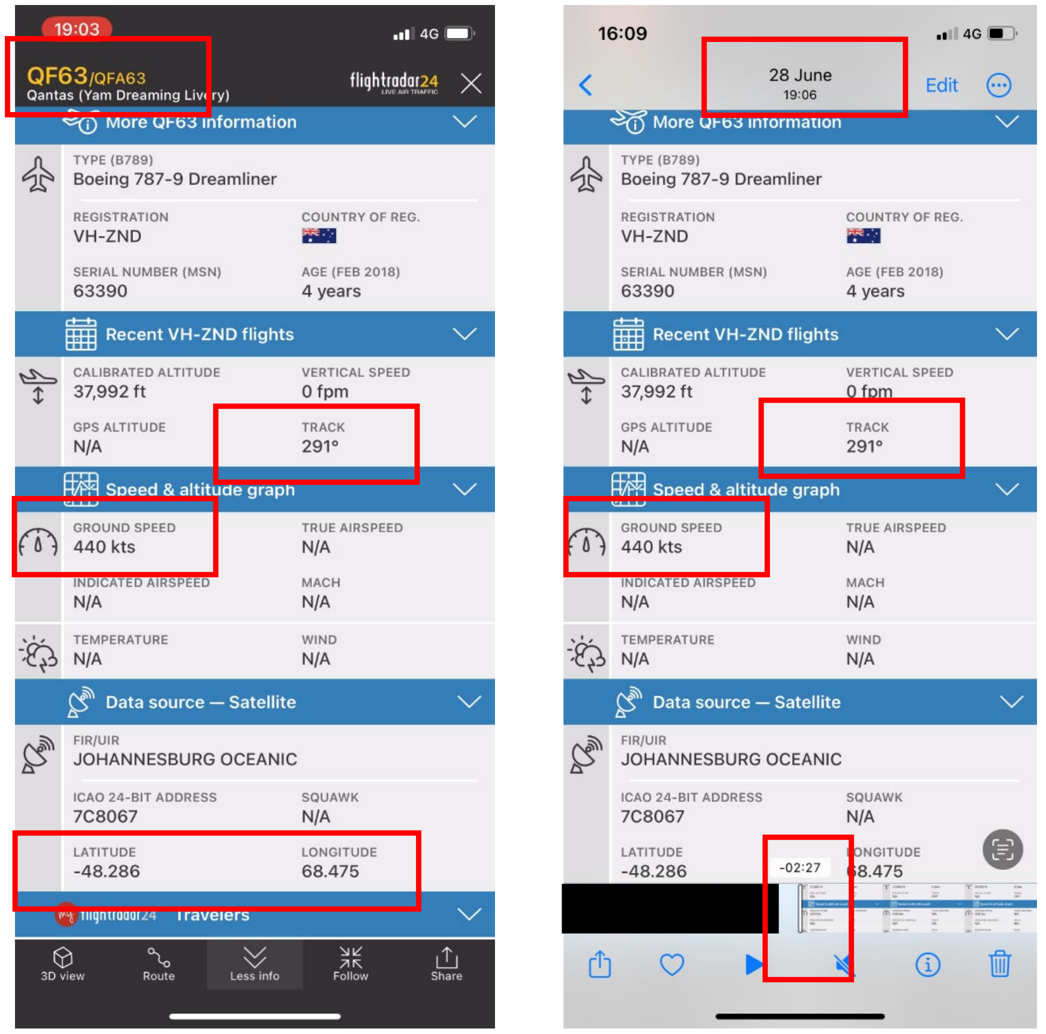
$$Track(0 < \theta < 90) = \tan^{-1}\left(\frac{\Delta Longitude}{\Delta Latitude}\right)$$

Bearing Track 0 < θ < 90 (dec degs)	Track error
23.22	0.22

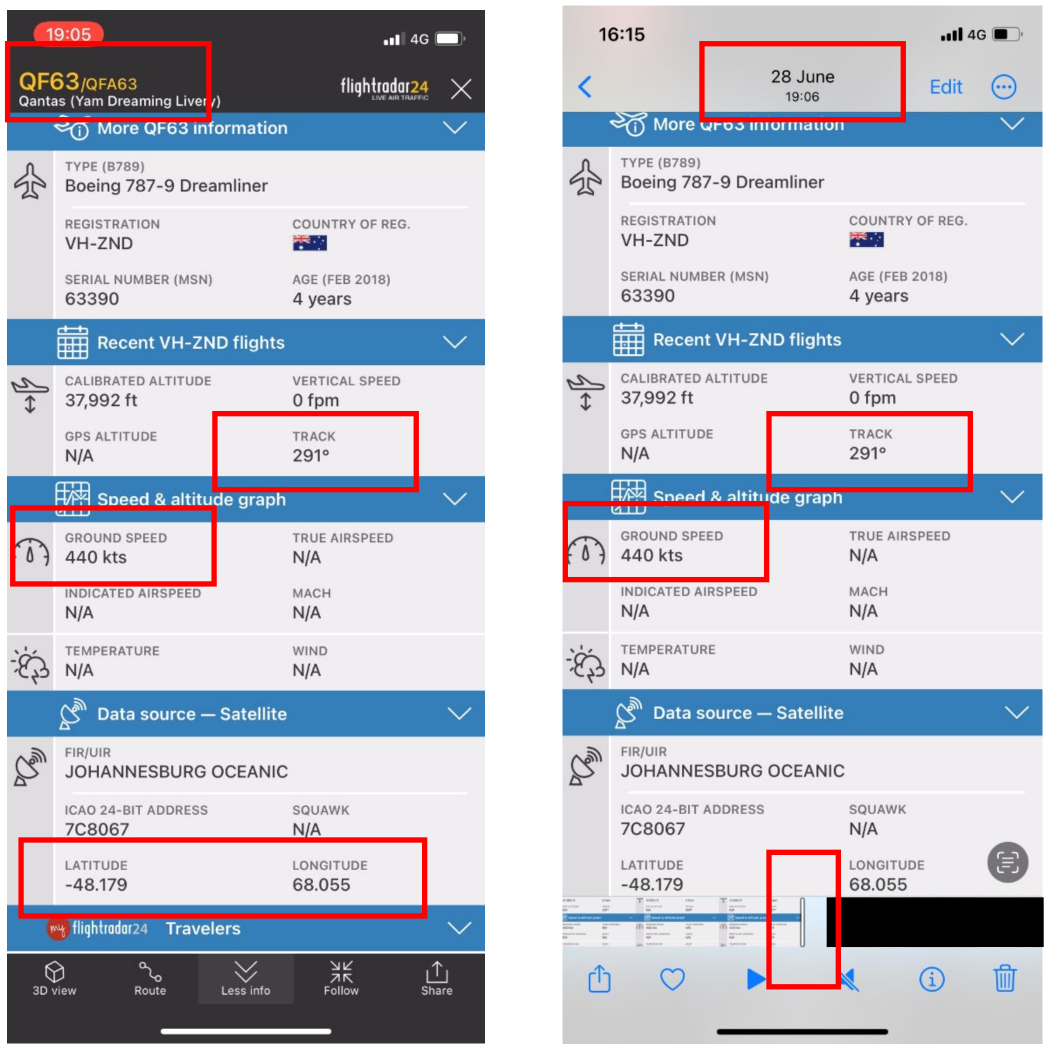
***Method* 4**

Flight travelling in
***any** direction and*
***anywhere** on the Earth*

Spherical Trigonometry



Data at the ***START*** *of recording*



Data at the ***END*** *of recording*

$Method\ 4: Spherical\ trigonometry$

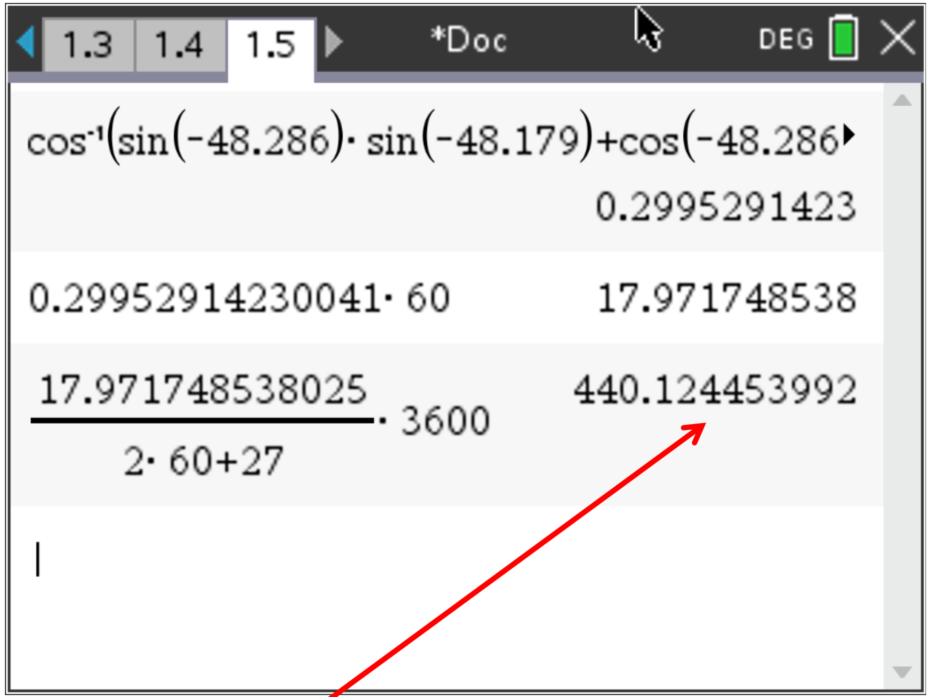
$Calculating\ the\ angle, distance\ \&\ speed$

Date	2022-06-28
Flight Number	QF63
From	Sydney SYD
To	Jo’burg JNB
Latitude1 (degrees)	-48.286
Longitude1 (degrees)	68.475
Latitude2 (degrees)	-48.179
Longitude2 (degrees)	68.055
ΔTime (mm:ss)	02:27
Speed (knots)	440
Track (˚T)	291

$arc\ angle(°)$

$distance(nm)$

$speed(knots)$



$Speed\ \approx 440\ knots$

$$arc\ angle = \cos^{-1}[\sin(Lat_1) \cdot \sin(Lat_2) + \cos(Lat_1) \cdot \cos(Lat_2) \cdot \cos(|Long_2 - Long_1|)]$$

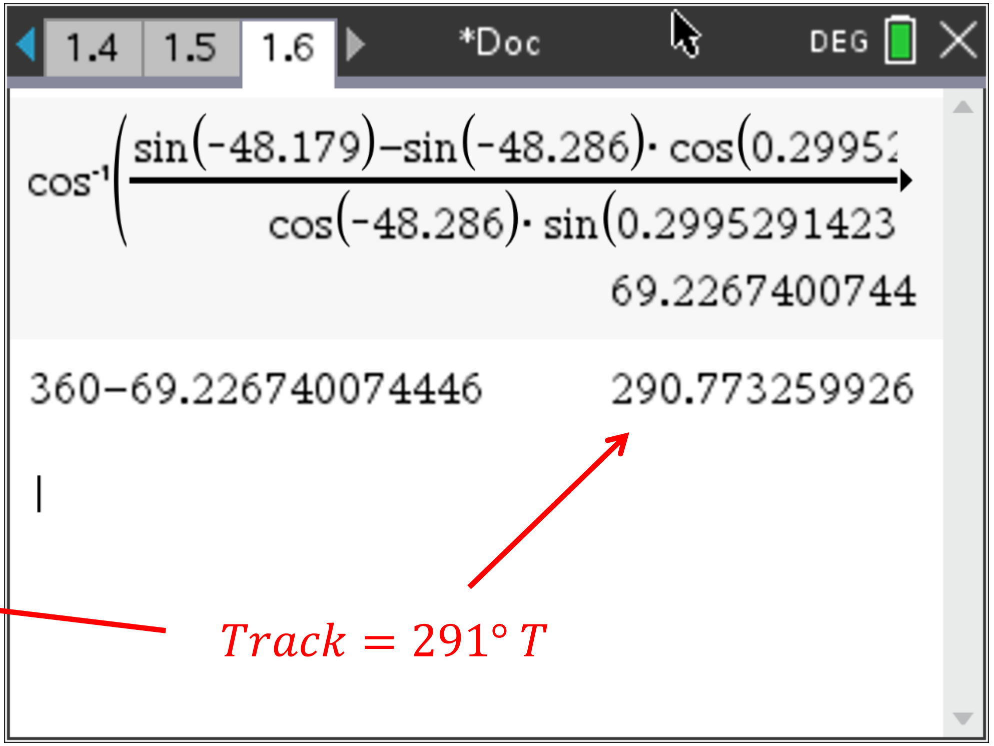
$Method\ 4$: $Spherical\ trigonometry$ — $Calculating\ the\ track$

Date	2022-06-28
Flight Number	QF63
From	Sydney SYD
To	Jo'burg JNB
Latitude1 (degrees)	-48.286
Longitude1 (degrees)	68.475
Latitude2 (degrees)	-48.179
Longitude2 (degrees)	68.055
ΔTime (mm:ss)	02:27
Speed (knots)	440
Track (°T)	291

$$\theta° = \cos^{-1}\left(\frac{\sin(Lat_2) - \sin(Lat_1)\cos(arc\ angle)}{\cos(Lat_1)\sin(arc\ angle)}\right)$$

$\theta° =$

$Track\ correction$



$Track = 291°\ T$

$If\ Lon_2 > Lon_1\ then\ Track = \theta°$ ✗

$If\ Lon_2 < Lon_1\ then\ Track = 360° - \theta°$ ✓

Data from Flightradar24	Edit only the red cells	Blue cells are calculated

10							
Date:	2022-06-28		4G:	✓		✓	✗
Flight No:	QF63		Satellite:	✓			
From:	SYD	Sydney	Hemisphere:	SE			
To:	JNB	Johannesburg	Direction:	NW			
Altitude:	36000	feet	Method:	4			
Altitude:	10972.8	metres					

ΔT ('mmss)	ΔT (seconds)	Latitude1 (±decimal)	Longitude1 (±decimal)	Latitude2 (±decimal)	Longitude2 (±decimal)	Speed (knots)	Track (° True)
0227	147	-48.2860	68.4750	-48.1790	68.0550	440	291

	C = 40 000	r = 6371							
Calculated angle a (degrees)	Calculated distance (nm)	Calculated distance (nm)	Calculated speed (knots)	Calculated speed (km/h)	Calculated course (W) (° True)	Calculated course (E) (° True)	Speed error (knots)	Speed error (%)	Track error (degrees)
0.299529142	17.970	17.984	440.1	815.0	290.8		0.1	0.0%	-0.2

$$a = \cos^{-1}[\sin(Lat_1)\sin(Lat_2) + \cos(Lat_1)\cos(Lat_2)\cos(|Long_2 - Long_1|)]$$

$$C = \cos^{-1}\left[\frac{\sin(Lat_2) - \sin(Lat_1)\cos(a)}{\cos(Lat_1)\sin(a)}\right]$$

Who said trigonometry is useless!!!

Navigation is a very practical application of trigonometry.

Thank you.

The End

References:

Bowditch, N. and United States National Geospatial-Intelligence Agency (2013). *The American Practical Navigator*. New York, NY: Skyhorse.

Wikipedia Contributors. (2020, October 13). Great-circle distance. Retrieved November 2, 2020, from Wikipedia website: https://en.wikipedia.org/wiki/Great-circle_distance#:~:text=The%20great%2Dcircle%20distance%2C%20orthodromic,line%20through%20the%20sphere’s%20interior).

Enzo Vozzo

After working as a Technical Officer at Telstra, Enzo graduated from Monash University in 2005 with a Bachelor of Technology (Computer Studies) and taught Electronics and Communications Engineering at Chisolm TAFE.

In 2013 he graduated from RMIT University with a Graduate Diploma of Education teaching Secondary School Mathematics and Science.

Since 2016 he has been teaching Mathematics at Mentone Grammar.

MENTONE GRAMMAR

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$$i = \sqrt{-1} \qquad \phi = \frac{1+\sqrt{5}}{2}$$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} \qquad \pi = 4\int_0^1 \sqrt{1-x^2}\,dx$$

YouTube Channel: Maths Whenever:
https://www.youtube.com/channel/UCFLdfe_y2OQ1MZvGjha9taQ/videos